The Electrical Properties of High Permeability Wires Carrying Alternating Current

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I—Introduction

It is well known that certain alloys of nickel and iron of the " permalloy " and " mumetal " group have abnormally high values of magnetic permeability in small fields, and the properties of these alloys have been the subject of a considerable amount of research during the past ten or twelve years. It seems to have escaped attention that their high magnetizability at low field-strengths causes wires of these alloys to have many interesting properties when used as conductors of alternating current.

Owing to the " skin effect ", the A.C. resistance of a wire bears to its D.C. resistance a definite ratio which is a function of the frequency of the current, and of the diameter, the electrical conductivity, and for ferro-magnetics the permeability of the wire. With fine wires (diameter 0·5 mm. or less) of non-magnetic material such as copper, this ratio does not materially exceed unity until the frequency approaches 10^5 cycles per second. But with wires of ferro-magnetic alloys, whose permeability may be in the neighbourhood of 50,000, the ratio R_{AC}/R_{DC} is appreciably greater than unity even at the lowest audio-frequencies. Furthermore, a ferro-magnetic wire possesses an internal self-inductance, which, with alternating current, is also a function of the frequency, diameter, conductivity, and permeability. Now permeability is a function of magnetization, and since a wire carrying A.C. is magnetized by the current as well as by the external field in which it may be situated, it may be expected that its A.C. resistance and inductance will show some sort of dependence on both current and external field.

In this paper an account is given of an investigation of these phenomena, of which perhaps the most striking is the very large change of A.C. resistance which occurs when an external magnetic field of the order of that of the earth is applied to a wire in the direction of its axis. A letter describing briefly this change of effective resistance was published in June, 1935.* The resistance change is many times greater than the corresponding magneto-resistance change with direct current, which, as is well known, is itself remarkably large in these alloys.

II—Experimental Details

It is necessary to remark at the outset that the material used in these experiments is one or other of the high permeability alloys of nickel and iron of range near to 78% nickel prepared in the form of wire, usually of 26 standard wire gauge, and annealed in some special way. Since the particular alloy used and the heat treatment given to it have an important effect on the character and magnitude of the observed phenomena, § VII is devoted to these details. For the moment it is enough to state that in the experiments described below all the material has been subject to some quite definite annealing process, which at present may conveniently be left unspecified.

Apparatus—A valve oscillator was used as source of alternating current in all the measurements. This instrument provides a reasonably pure wave form if the frequency is not too low and if the iron core in the oscillator inductance is removed; to the degree of accuracy aimed at in these experiments its small harmonic content did not seriously affect the results. Since, however, the impedance varies with the frequency of the A.C. supply, it is evident on general grounds that in making quantitative measurements of the highest degree of accuracy, which shall also be strictly repetitive, it will be essential to use a really pure sine wave oscillator.

A very convenient bridge for the determination of the resistance and inductance of various specimens of wire is a modified Kelvin double bridge whose circuit is shown in fig. 1. P, Q, and R_1 are non-inductive resistances, R_1 being variable, R, L are the resistance and inductance of the nickel iron wire, MA is a thermomilliammeter of resistance r, and C is a variable capacity.

The indicator is a Campbell vibration galvanometer VG.

The conditions for balance are:—

$$R = QR_1/P$$

$$L = CQ\left[R_1 + \frac{rP}{P+Q+r}\right],$$

and it was generally found convenient to make P = 100 ohms, Q = 1 ohm.

* Harrison, Turney, and Rowe, ' Nature,' vol. 135, p. 961, 8 June (1935).

To find the current i in the specimen in terms of the current i_1 observed in the ammeter, we have

$$i = i_1\left[1 + \frac{r}{P + Q}\right].$$

By introducing a battery instead of the oscillator and an ordinary galvanometer at VG, D.C. resistances can also be measured.

The accuracy with which the instrumental measurements can be made is such that at a frequency of 500 cycles/second, the effective resistance is determined to one-tenth of 1% and the inductance to one-fifth of 1%.

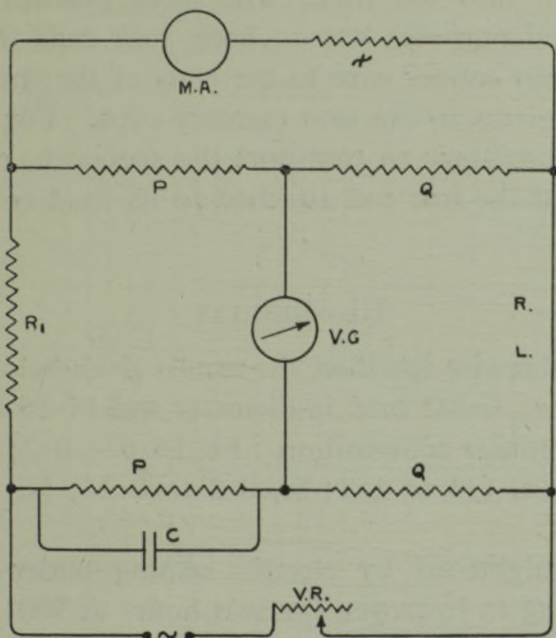

FIG. 1—Kelvin double bridge adapted to A.C. measurements.

No particular precautions were taken in measuring the absolute value of the frequency, but variations did not exceed ½-cycle/second.

In order to apply a uniform longitudinal field to a wire, two alternative methods were used, a pair of Helmholtz coils, or a solenoid long compared with the wire.

Method of Mounting the Wires—The effect of strain on the properties with which we are concerned has been found to be extremely important. The relations between strain and impedance have not been investigated, but it became evident that it was vital to eliminate both strain and temperature variations in order to ensure that all impedance measurements should be repetitive within the limits of accuracy determined by the character of the A.C. supply, and the nature of the bridge. Constancy of temperature was easy to attain, but much experimental work was necessary before a satisfactory way was achieved of mounting isolated wires so as to avoid strain. As a result of these experiments two necessary conditions emerged:—

(i) The wire must, if horizontal, be supported along its length; in other words it should lie along a plane surface, or be embedded in some form of vaseline or petroleum jelly.

(ii) One end of the wire only may be fixed by solder or otherwise to a copper or brass terminal on the base plate; the other end must be as nearly free from constraint as may be, consistent with the necessity of conducting current into the wire. The most practicable method for purely experimental purposes was to keep *both* ends free by soldering short lengths of thin copper wire to the ends of the specimen, allowing these copper end pieces to dip into mercury cups. For other purposes, where it may be necessary to transport the specimen, one end may be fixed by solder and the free end attached to its fixed contact piece by a fine copper spiral.

III—Results

Except where otherwise specified, the results given below were obtained with a mumetal wire, 0·445 mm. in diameter and 15·25 cm. long, having the following percentage composition : Fe, $16 \cdot 6 \pm 0 \cdot 2$; Cu, $5 \cdot 0 \pm 0 \cdot 1$; Mn, $0 \cdot 5 \pm 0 \cdot 1$; Cr, 1·4 to 1·8; Si, 0·2 to 0·25; Ni, 77·3% by subtraction.

After being straightened by electric heating under tension it was annealed by heating in hydrogen for two hours at 900° to 920° C. and cooling slowly with the furnace.

Zero Longitudinal Magnetic Field—The wire, lying horizontally at right angles to the magnetic meridian, was demagnetized, and measurements were made of its effective resistance and reactance at various frequencies and with various currents. Some results are shown graphically in fig. 2. The outstanding feature of the curves at each frequency is the maximum through which the effective resistance passes as the current is varied; for convenience the current value at which the maximum occurs will throughout this paper be termed the " optimum current " for the wire in question in the prevailing conditions. Effective resistance and optimum curren both show a regular increase with increasing frequency, and these characteristics have been found in all the wires investigated, though the actual values vary widely according to the composition of the wire and the heat treatment it has received.

The reactance also has a maximum value for each frequency, in the case shown in fig. 2 at a value of current less than the optimum. But this is by no means an invariable characteristic, and the reactance curves are

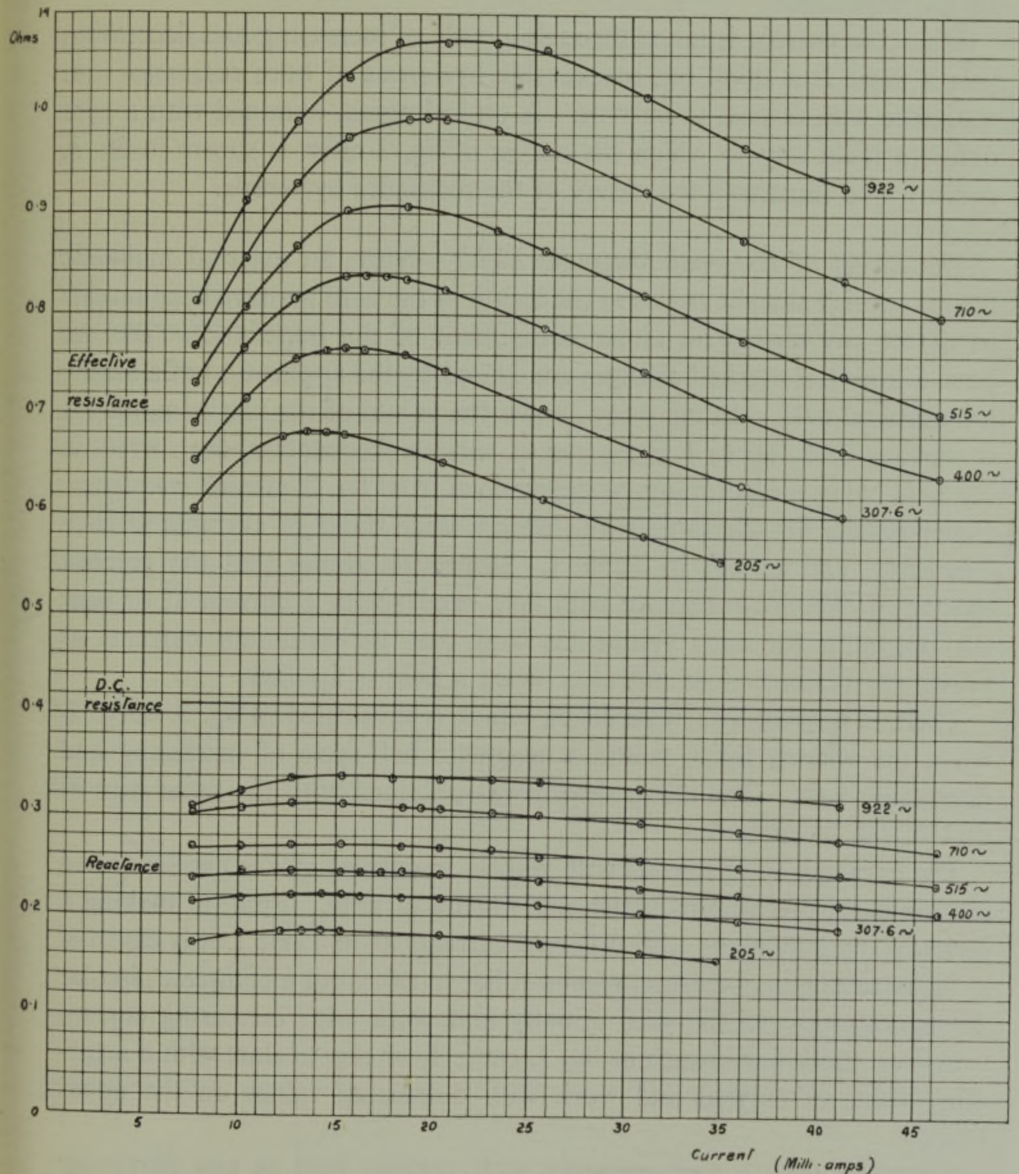

FIG. 2—Variation of effective resistance and reactance with current at different frequencies. Wire 15·25 cm. long, 0·445 mm. diameter.

found to differ considerably in character as the composition of the alloy is varied. For some alloys the maximum reactance occurs at a current above the optimum; in others a minimum occurs near the optimum current, followed by a maximum at a much higher value. Since, however, the reactance component is in general only about one-third of the effective resistance, these irregularities have no very pronounced effect on the impedance.

When the current is taken through a cycle of increasing and decreasing values there is a small but measurable hysteresis of both resistance and reactance with respect to current. A typical result, obtained with a wire different from that to which the curves of fig. 2 refer, is shown in fig. 3, curve 1, and illustrates the nature of the resistance hysteresis.

The Effect of External Magnetic Fields—When an external magnetic field is applied at right angles to the length of the wire we have been unable to detect any change in either component of the impedance. But we have found that marked changes are caused by the application of a steady longitudinal field. In fig. 4 are shown the values of the two components in various fields up to 1 gauss and at various frequencies, the current in the wire being the optimum, as determined from fig. 2, at each frequency. Fuller results for the same wire, using a wide range of currents at each frequency, are set out in Table I. Inspection shows that the greatest change of effective resistance with field occurs with a current at or near the optimum for each frequency, and that the optimum current, as previously defined, does not alter when a field is applied.

There is a small hysteresis effect, which is illustrated in fig. 3, curve 2, when the wire is taken round a magnetic half-cycle of increasing and decreasing field. The curve shown was obtained with the same wire as that of fig. 3, curve 1.

The existence of hysteresis with respect to current makes necessary two experimental precautions which are worthy of note. First, in making a series of measurements with different currents as in Table I, it is essential in order to obtain consistent results either to carry out a complete circular demagnetization of the wire initially and work with increasing current values on the lower branch of fig. 3, curve 1, or else to start with a saturating value of current and work with decreasing values on the upper branch.

Secondly, during measurements with varying field at any particular current value, when the field is increased by a step the effective resistance of the wire decreases. To restore balance the variable arm of the bridge must also be decreased, and since the ratio is 100/1 this leads to an appreciable increase of current above the chosen value. If we are working with increasing current values (the first method described above) the subsequent restoration of the current to its chosen value will not, owing

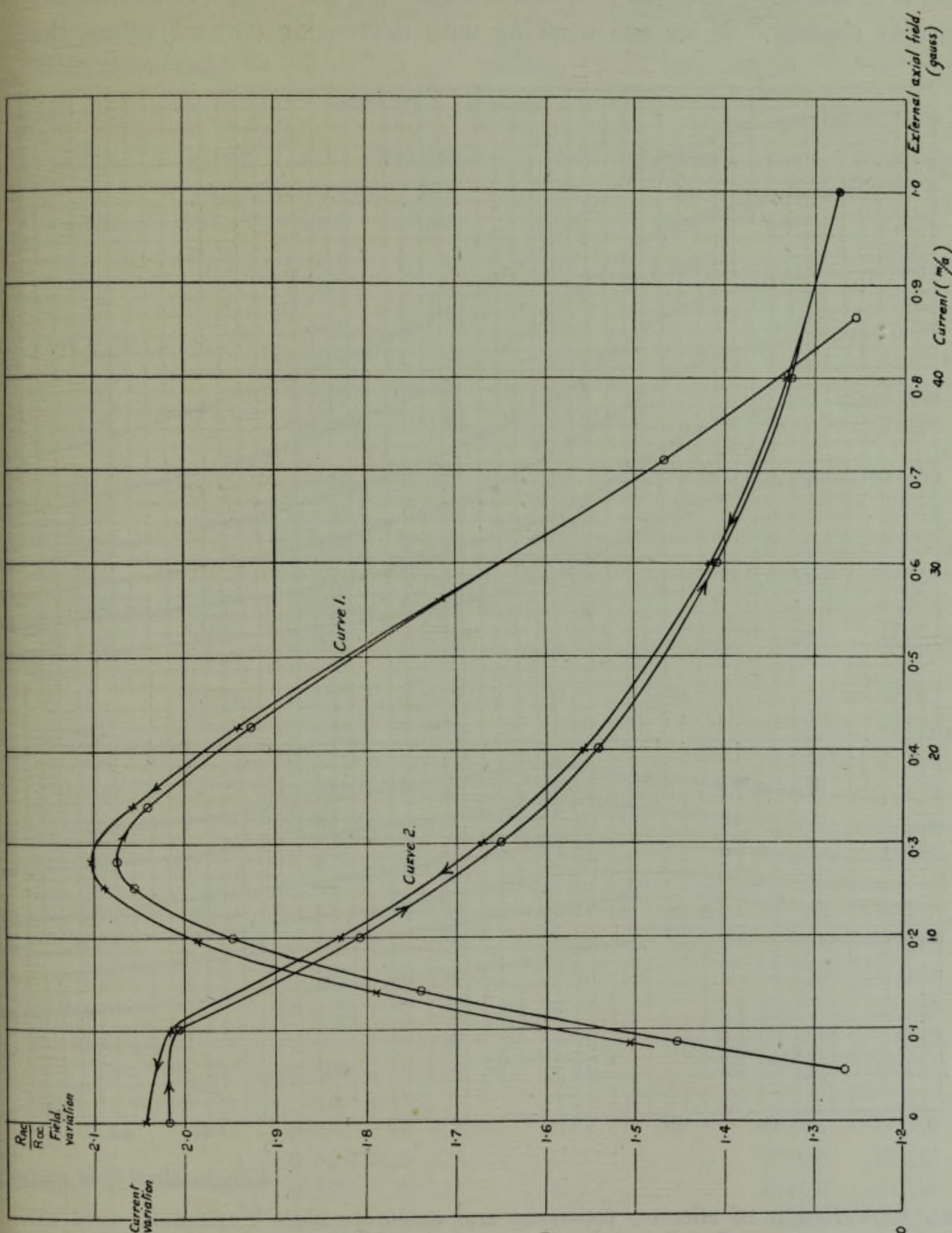

FIG. 3.—Mumetal wire: length 23·5 cm.; diameter 0·0457 cm.; frequency 500 pp.s. ⊙ = increasing field and current; × = decreasing field and current.

to hysteresis, restore the wire to its original state of circular magnetization, and it is therefore necessary before re-balancing the bridge to reduce the current slightly. If we are working with decreasing current values the

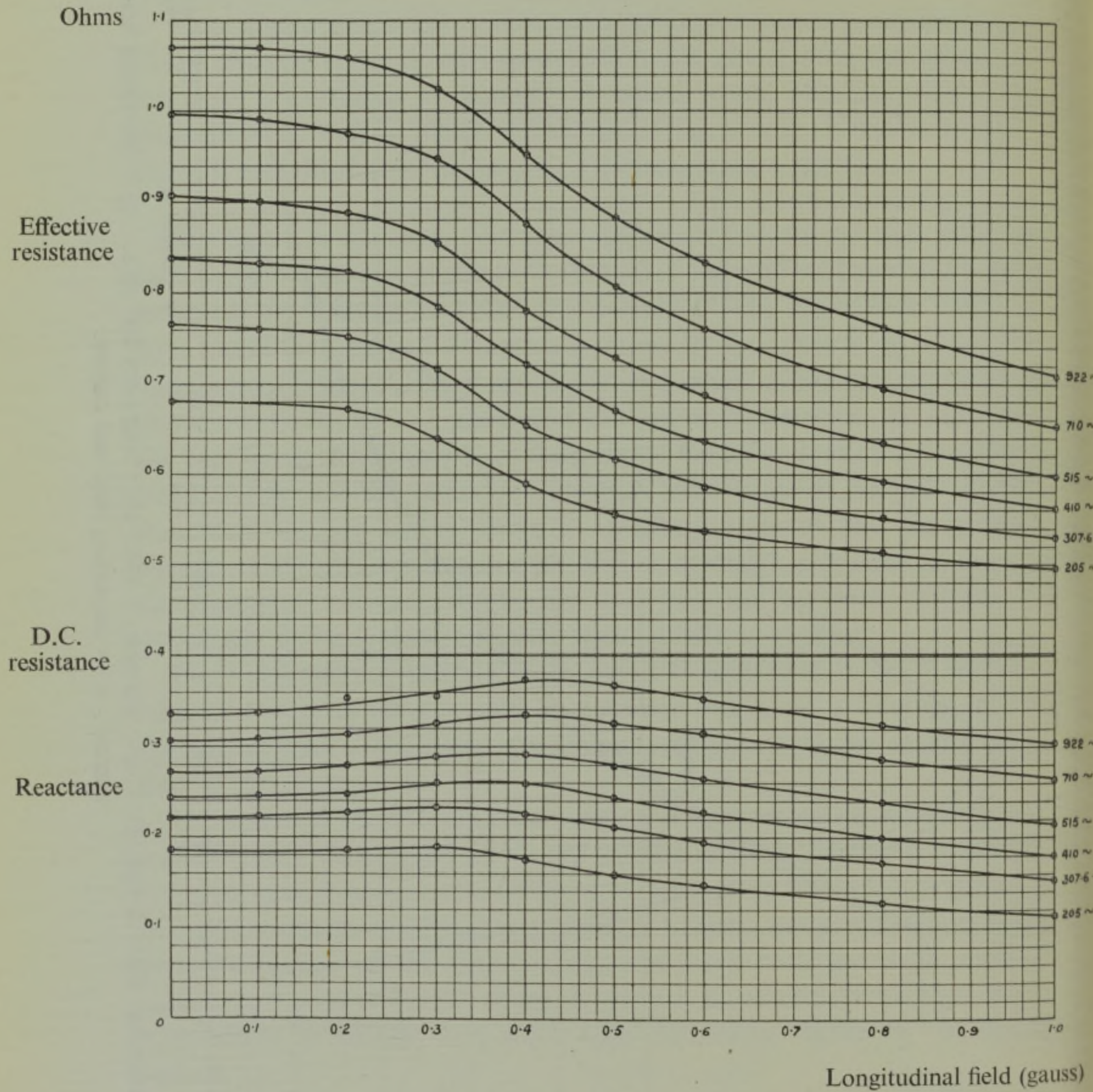

FIG. 4—Variation of effective resistance and reactance with longitudinal field at different frequencies and optimum current. Wire 15·25 cm. long; 0·445 mm. diameter.

need for this precaution does not arise, and this was the method actually employed in obtaining the curves of fig. 4.

Wires of Different Dimensions—With wires of different lengths, but

Table I—The variation of A.C. resistance and reactance with current and applied longitudinal field at constant frequency

Wire from heat 94—0·02225 cm. radius, 15·25 cm. length, 0·410 ohm D.C. resistance; frequency 515~

Field in gauss	Current in m/A.	A.C. resistance ohms	Reactance in ohms	Field in gauss	Current in m/A.	A.C. resistance ohms	Reactance in ohms
0	170·7	0·478	0·131	0	150·7	0·489	0·141
0·2		0·475	0·131	0·2		0·485	0·141
0·4		0·469	0·129	0·4		0·477	0·139
0·6		0·462	0·126	0·6		0·468	0·135
0·8		0·454	0·121	0·8		0·459	0·130
1·0		0·449	0·116	1·0		0·452	0·124
0	135·6	0·500	0·150	0	119·0	0·514	0·162
0·2		0·495	0·148	0·2		0·509	0·161
0·4		0·486	0·146	0·4		0·496	0·159
0·6		0·476	0·144	0·6		0·484	0·155
0·8		0·466	0·138	0·8		0·472	0·147
1·0		0·458	0·132	1·0		0·462	0·139
0	101·2	0·533	0·172	0	91·0	0·551	0·182
0·2		0·526	0·173	0·2		0·543	0·184
0·4		0·511	0·172	0·4		0·524	0·182
0·6		0·495	0·168	0·6		0·505	0·178
0·8		0·481	0·161	0·8		0·490	0·170
1·0		0·471	0·151	1·0		0·476	0·159
0	80·9	0·573	0·192	0	70·8	0·598	0·205
0·2		0·562	0·195	0·2		0·586	0·209
0·4		0·542	0·194	0·4		0·558	0·210
0·6		0·519	0·190	0·6		0·530	0·204
0·8		0·499	0·180	0·8		0·507	0·190
1·0		0·484	0·166	1·0		0·491	0·175
0	60·7	0·635	0·217	0	50·6	0·681	0·230
0·2		0·621	0·222	0·2		0·666	0·237
0·4		0·584	0·225	0·4		0·621	0·244
0·6		0·549	0·218	0·6		0·573	0·234
0·8		0·521	0·200	0·8		0·540	0·214
1·0		0·502	0·186	1·0		0·520	0·196
0	45·5	0·708	0·237	0	41·0	0·743	0·245
0·2		0·696	0·246	0·2		0·726	0·255
0·4		0·639	0·254	0·4		0·664	0·264
0·6		0·589	0·242	0·6		0·616	0·251

TABLE I—(continued)

Field in gauss	Current in m/A.	A.C. resistance ohms	Reactance in ohms	Field in gauss	Current in m/A.	A.C. resistance ohms	Reactance in ohms
0·8	45·5	0·549	0·217	0·8	41·0	0·565	0·226
1·0		0·527	0·200	1·0		0·540	0·204
0	35·9	0·778	0·253	0	30·8	0·822	0·260
0·2		0·764	0·262	0·2		0·808	0·272
0·4		0·690	0·274	0·4		0·722	0·283
0·6		0·623	0·258	0·6		0·644	0·265
0·8		0·579	0·232	0·8		0·598	0·238
1·0		0·552	0·210	1·0		0·566	0·214
0	25·6	0·866	0·264	0	20·5	0·902	0·270
0·2		0·856	0·276	0·2		0·892	0·275
0·4		0·757	0·292	0·4		0·781	0·292
0·6		0·669	0·268	0·6		0·687	0·266
0·8		0·615	0·239	0·8		0·633	0·239
1·0		0·581	0·214	1·0		0·596	0·215
0	18·5	0·904	0·271	0	15·4	0·894	0·271
0·2		0·895	0·276	0·2		0·883	0·275
0·4		0·786	0·289	0·4		0·778	0·287
0·6		0·688	0·263	0·6		0·683	0·260
0·8		0·635	0·238	0·8		0·630	0·234
1·0		0·597	0·214	1·0		0·593	0·211

otherwise identical, it has been found, as is natural, that the optimum currents and the ratios,

$$\frac{\text{A.C. effective resistance}}{\text{D.C. resistance}},$$

in zero field do not vary but that the effect of applying longitudinal fields increases with the length, owing to the reduction in self-demagnetization. This is shown in the graphs of fig. 5 which were obtained by cutting down a 26 S.W.G. wire originally 41 cm. long.

In wires of different diameter, on the other hand, there are differences not only in the longitudinal self-demagnetization but also in the distribution of current over the cross-section, so that their A.C. properties are fundamentally different. In investigating the variations in these properties with diameter an experimental difficulty arises.

It will be shown in § VII that heat treatment has a very important effect on the characteristics of a wire of any particular composition. In a batch of wires having the same length, diameter, and composition all subjected to the same heat treatment, there is always found to be a gradation of quality, so that no two wires of the same batch are strictly alike magnetically. It is not, therefore, an easy matter to devise a satisfactory way of making comparable measurements on wires drawn

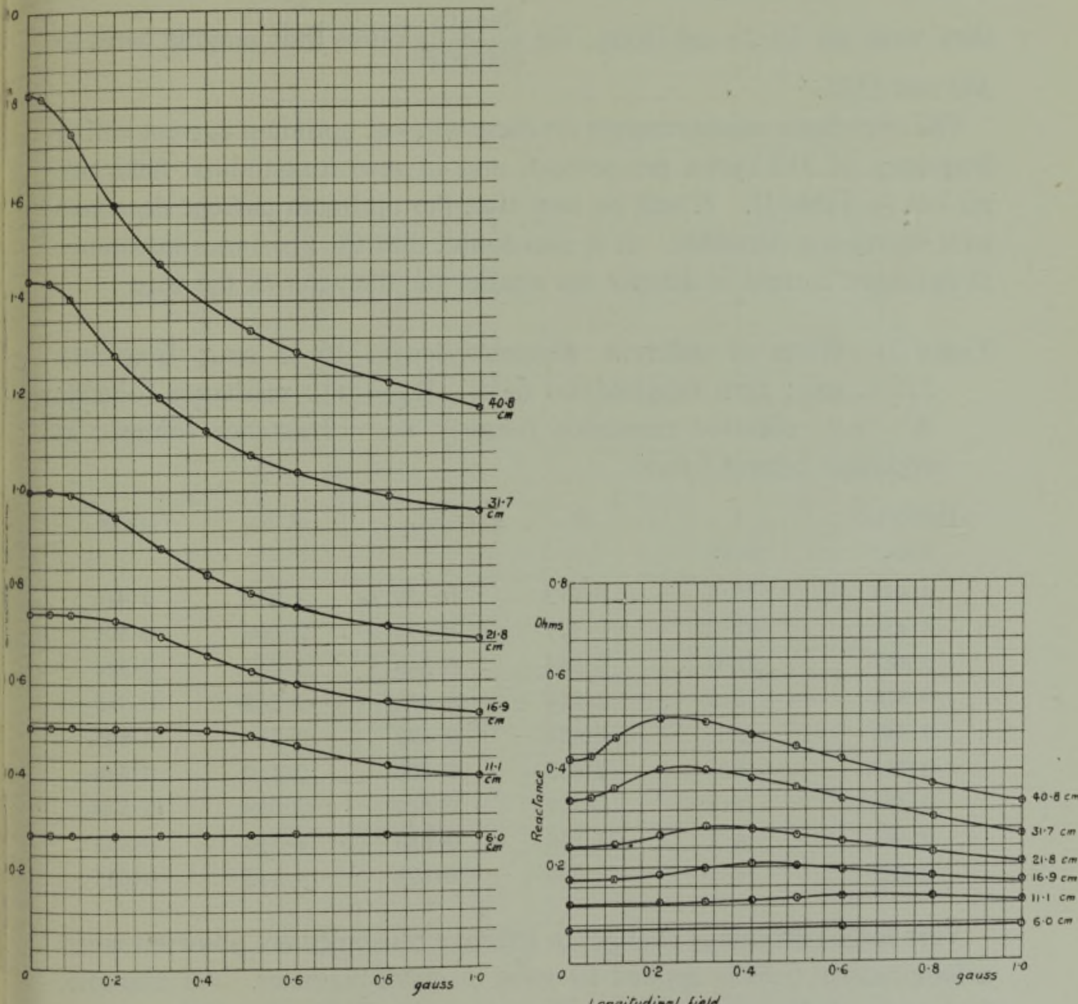

FIG. 5—Variation of resistance and reactance with longitudinal field. Wires of diameter 0·445 cm., and different lengths. Frequency 515~. "Optimum current" 25 m/A.

to different diameters and subsequently annealed, even though annealed in the same "heat", the less so since the effect of cold working on a wire previous to its annealing has a marked and uncontrollable influence upon its magnetic properties.

The best way hitherto discovered for producing comparable wires of different diameter is to choose specimens of the same length and diameter from one heat-treated batch which differ as little as may be in their magnetic properties, and to etch them with an acid mixture for different lengths of time. By this means we obtained a series of wires having as nearly as possible the same physical characteristics and differing only in diameter. Their diameters lay between 0·445 mm. and 0·1 mm. and they were all 15·25 cm. long, the $\frac{\text{length}}{\text{diameter}}$ ratios thus ranging between 343 and 1525.

The impedance measurements on these wires at optimum current with a frequency of 515 cycles per second, and in zero longitudinal field, are set out in Table II. It will be seen that the optimum current decreases with decreasing diameter. It is also found that the resistance-maximum at optimum current is sharper the smaller the diameter of the wire.

TABLE II—Wires of different diameter, length 15·25 cm.; frequency 515 c./sec.; zero longitudinal field. R_0 = D.C. resistance (ohms); R = A.C. effective resistance (ohms); ωL = reactance (ohms); at optimum current I m/A.

Diameter mm.	I m/A.	R	ωL	R/R_0	$\omega L/R_0$
0·445	25·5	0·713	0·159	1·870	0·416
0·437	22·5	0·777	0·195	2·000	0·501
0·427	21·0	0·864	0·240	2·067	0·574
0·376	12·5	1·218	0·412	2·248	0·761
0·346	9·0	1·433	0·498	2·316	0·805
0·315	8·5	1·615	0·506	2·150	0·674
0·264	5·5	2·231	0·624	2·039	0·570
0·183	4·2	3·558	0·830	1·510	0·352
0·102	3·5	7·278	0·623	1·115	0·095

Very large percentage changes in effective resistance are observed when a longitudinal field is applied to wires of small diameter. An example, for a wire of diameter 0·183 mm., is shown in fig. 6. It will be noticed that the resistance changes by about 20%, or 0·6 ohm, when the field is reduced from 0·18 gauss to zero, so that a change of this magnitude is produced by simply rotating a horizontal wire through 90° in the earth's magnetic field.

The Temperature Coefficient—The temperature coefficient of effective resistance was measured for the wire from which the curves of figs. 2 and 4 were obtained, using the optimum current at 515 cycles per second, and with longitudinal fields between zero and 1 gauss. The relation

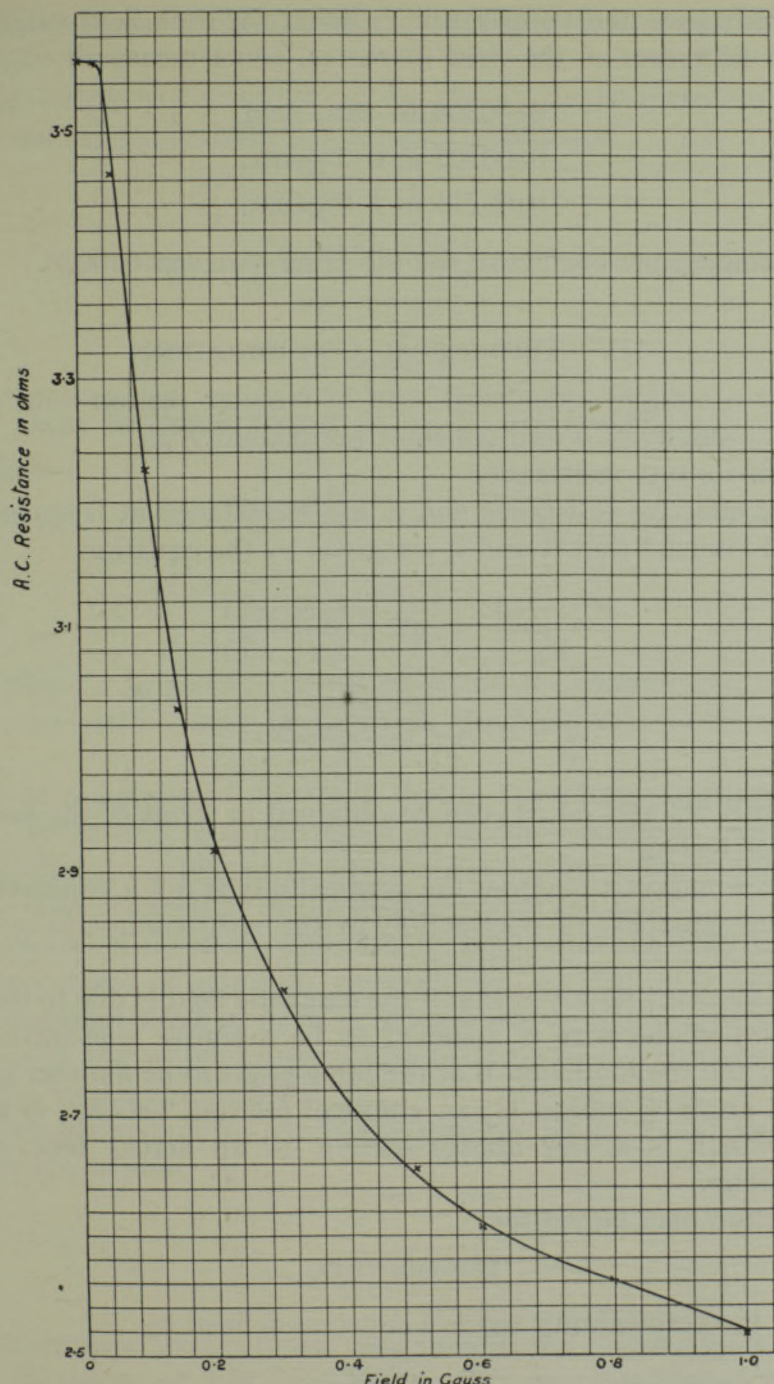

FIG. 6—Variation of A.C. resistance with applied longitudinal field. Frequency 515~, optimum current. Length 15·25 cm.; diameter 0·0183 cm.

between resistance and temperature is linear for each field value, but the coefficient decreases as the field increases, some measured values being

0·00306 in zero field.

0·00303 in 0·2 gauss.

0·00211 in 1·0 gauss.

The D.C. temperature coefficient for the same wire was 0·0026.

IV—Theoretical Considerations

Consider a straight cylindrical wire of circular section with radius a, electrical conductivity σ and constant permeability μ_0.

Let R_0 be its D.C. resistance.

R be the effective resistance and L the inductance at frequency $\omega/2\pi$.

It is well known* that R and L are given by the equation

$$\frac{R + j\omega L}{R_0} = \sqrt{2}\,\xi\,\frac{J_0(\xi\sqrt{8})}{J_1(\xi\sqrt{8})}, \tag{1}$$

where

$$\xi = a\sqrt{\frac{-\pi\mu_0\omega\sigma j}{2}}, \tag{2}$$

from which R/R_0 and $\omega L/R_0$ can be obtained in terms of the *ber* and *bei* functions of Kelvin.

With ferro-magnetic material the permeability is not a constant and the fundamental equation of conduction in the wire cannot in general be solved.

It is well known,† however, that if the magnetic field H due to the current at any point in the wire is assumed to be a simple sine-wave without harmonics, and the induction B at the same point to be another sine-wave lagging an angle ψ behind H, an elliptical relation between B and H is obtained which is a fair approximation to an actual ferro-magnetic hysteresis loop.

The relation may be written

$$B = \mu\varepsilon^{-j\psi} H, \tag{3}$$

where μ is the permeability and is a constant defined by

$$B_0 = \mu H_0, \tag{4}$$

* Zenneck, 'Ann. Physik,' vol. 2, p. 1135 (1903).

† Russell, "Alternating Currents," vol. 2, chap. x; Gall and Sims, 'J. Instn. elect. Engrs.', vol. 74, p. 453 (May, 1934).

B_0 and H_0 being the maximum values of induction and field respectively in the ellipse.

For the simple permeability μ_0 in (2) we must therefore in ferromagnetic material substitute the complex expression

$$\mu_0 = \mu\varepsilon^{-j\psi}$$
$$= \mu_1 (1 - jb), \qquad (5)$$

where

$$\mu_1 = \mu \cos \psi, \qquad (6)$$

and

$$b = \tan \psi. \qquad (7)$$

It may easily be shown that if W be the hysteresis loss in the wire in ergs/cycle/cc.

$$W = \frac{\sin \psi}{4\mu} B_0^2 = KB_0^2,$$

where

$$K = \frac{\sin \psi}{4\mu}. \qquad (8)$$

Thus the angle ψ is given in terms of μ and the quantity K, which as a rough approximation may be taken as the Steinmetz coefficient of the material.

It must be remembered, however, that in a wire the value of H_0 varies from zero at the axis to a maximum at the circumference, and that with actual material μ as defined by (4) is not a constant, but varies with H_0 and is therefore a function of the distance from the axis.

But the solution (1) of the equation of conduction can only be obtained on the hypothesis of constant permeability, so that it is necessary to assume a constant value of μ, or in other words to take μ as a mean value averaged over the whole cross-section of the wire. The same probably applies also to the value of ψ.

With this understanding we can substitute for μ_0 in (2) the complex expression (5), so that

$$\xi^2 = -(b + j)\, k^2,$$

where

$$k = a \sqrt{\frac{\pi \mu_1 \omega \sigma}{2}}. \qquad (9)$$

This substitution makes it less easy than in the classical case to obtain from (1) the values of R/R_0 and $\omega L/R_0$, but the computation for a considerable range of values of k and b has been carried out by Wwedensky,*

* Wwedensky and Schillerow, 'Z. Physik,' vol. 34, p. 309 (1925); *see also* Mittelstrass, 'Arch. Elektrotech.,' vol. 18, p. 595 (1927); Ermolaev, *ibid.*, vol. 23, p. 101 (1929); Antik, *ibid.*, vol. 25, p. 125 (1931).

who has given his results in a convenient form as a mesh diagram in which values of R/R_0 and $\omega L/R_0$ are plotted as abscissae and ordinates respectively for various values of k and b, and curves of constant k and constant b drawn to form the mesh. In the present work R/R_0 and $\omega L/R_0$ have been measured, so that the corresponding values of k and b can at once be read off from the diagram. μ follows from (6), (7), and (9) for

$$\begin{aligned}\mu &= \frac{\mu_1}{\cos\psi}\\ &= \frac{2k^2}{\pi a^2\omega\sigma\cos\psi}\\ &= \frac{2R_0}{\omega L\cos\psi}\cdot k^2, \qquad (10)\end{aligned}$$

where l = length of wire.

K then follows at once from (8).

It may be noted that the conception that hysteresis losses in alternating fields are the cause of a lag between B and H, which makes the permeability a complex function, is essentially that of Arkadiew.*

The treatment outlined above avoids the introduction of the fictitious "magnetic conductivity" postulated by that worker, and as has been seen leads to a solution in terms of recognized properties of ferro-magnetic material.

V—Application to Experimental Results

i—*Axial Field Zero*—In Table III, *a* and *b*, are shown the values of μ and K computed from the measured values of R/R_0 and $\omega L/R_0$ for a particular wire at various currents and frequencies, with zero axial field. The wire was of mumetal, 23·5 cm. long and 0·457 mm. in diameter.

The values of μ at constant frequency, when plotted against current, are found to lie on smooth curves which resemble in form the ordinary μ-H curve of ferro-magnetic material. The variations in K with current are, on the other hand, somewhat irregular, and very much smaller than those of μ, the values being, in fact, appreciably constant over a wide range of current. By taking the maximum value of μ and its corresponding value of K at each frequency we obtain Table IV.

The variation with frequency suggests that in order to compare directly the values of μ and K obtained from impedance measurements with those which can be derived from magnetic measurements, it is necessary that

* 'Phys. Z.,' vol. 14, p. 928 (1913).

TABLE III

a—VALUES OF $\mu \times 10^{-4}$

Current m/A.	Frequency, cycles/sec. 55	100	150	210	350	500
3·0	2·09	1·98	1·55	1·58	1·52	1·26
5·0	3·02	2·80	2·48	2·21	2·00	1·60
7·5	4·18	3·85	3·29	2·93	2·49	1·96
10·0	4·85	4·20	3·82	3·38	2·84	2·25
15·0	4·24	3·80	3·55	3·41	3·01	2·50
20·0	3·30	3·14	3·10	3·01	2·80	2·44
30·0	2·35	2·20	2·30	2·26	2·22	2·06
35·0	2·07	1·94	2·03	2·07	1·98	1·91

b—VALUES OF $K \times 10^{6}$

Current m/A.	55	100	150	210	350	500
3·0	4·7	6·4	6·4	7·3	8·2	9·9
5·0	5·0	5·4	6·1	7·0	7·9	9·4
7·5	4·2	4·7	5·4	6·2	7·3	8·9
10·0	3·4	4·4	5·0	6·0	7·0	8·6
15·0	3·4	4·5	5·0	5·9	7·7	8·1
20·0	3·9	4·7	5·3	6·0	6·8	8·1
30·0	4·0	5·2	5·8	6·6	7·7	8·7
35·0	4·3	5·0	5·9	6·8	8·1	9·0

the frequency should be extremely small. If we extrapolate the results in Table IV to zero frequency we get the following values of μ and K:

$$\mu = 5{\cdot}8 \times 10^{4}$$

$$K = 1{\cdot}7 \times 10^{6-}.$$

TABLE IV—MAXIMUM VALUES OF μ AND CORRESPONDING VALUES OF K AT DIFFERENT FREQUENCIES

Frequency	$\mu \times 10^{-4}$	$K \times 10^{6}$
55	4·86	3·4
100	4·41	4·4
150	3·85	5·0
210	3·53	5·9
350	3·03	6·7
500	2·51	8·1

The B-H curve for the wire and hysteresis loops with various values of maximum induction B_0 were found by enclosing the wire in a solenoid and using the standard ballistic method of magnetic measurement. The self-demagnetization effect was eliminated, approximately at any rate, by applying the correction appropriate to cylindrical wires.

The maximum permeability found in this way was $8 \cdot 5 \times 10^4$ and the values of K calculated from the formula $W = KB_0^2$ are shown in Table V.

TABLE V

B_0	W (ergs/cc.)	$K \times 10^6$
3000	14·4	1·6
4600	30·0	1·42
4900	34·5	1·44

If it is borne in mind that μ as derived from impedance measurements is by definition an average value of permeability over a range of fields from zero upwards, and therefore essentially less than the actual maximum permeability in the wire, it appears that the two results are in reasonable concordance. A more exact comparison is a matter of great difficulty, since we have no knowledge as to how to arrive at the mean value of permeability which it is necessary to assume in order to solve the equation of conduction in the wire. It is possible, however, to give a general qualitative interpretation of the results shown in Table III, *a*.

The variation of μ with current, at constant frequency, clearly derives from the normal variation of permeability with field. For the value of μ for any current must be some sort of average value of permeability calculated over a range of field between zero and the field at the surface of the wire, which is proportional to the current. Thus, whatever method of calculation of the average be employed, the μ-current curve must have the same general form as the curve connecting permeability and field, namely a rather sharp rise to a maximum followed by a slower decrease.

As the frequency increases, the total current in the wire remaining constant, the distribution of the current over the cross-section changes so that it is concentrated more and more in the outer layers, with the result that the field at points within the wire becomes smaller, although that at the surface is unchanged since it depends only on the total current. If in fig. 7, curve 1 represents the distribution of permeability from the axis to the circumference of the wire (of radius *a*) at some given low frequency, the corresponding distribution for the same current at a higher frequency will be given by a curve of the form of curve 2. The average permeability over the cross-section, or μ, will evidently be smaller when calculated from curve 2 than from curve 1, particularly for small or moderate current values. When the current is so large that the field at the surface approaches saturation, and the permeability there is small, it can be easily seen that the average values of permeability at different frequencies will tend to equality. It is also easy to see that the maximum values of μ (the average permeability) will decrease and will occur at higher values of current as the frequency increases. Reference to Table III, *a*, shows that these are the salient features of the experimental results, which can thus be broadly and qualitatively interpreted in terms of the ordinary magnetic properties of the wire.

In attempting a quantitative explanation it must be remembered first of all that we have assumed that at any point in the wire the hysteresis loop due to a single cycle of current is an ellipse.

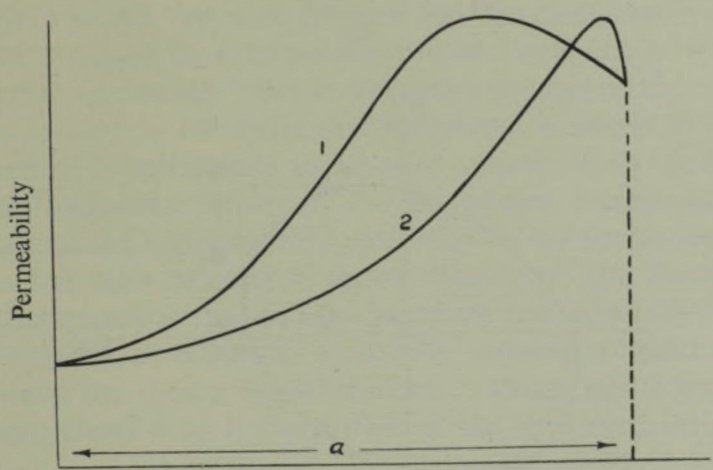

FIG. 7.

Let fig. 8 represent one such ellipse, at a point distant *r* from the axis, with a maximum value of field H_0 and of induction B_0. The permeability at this point is defined as $\mu_r = \dfrac{B_0}{H_0} = \dfrac{PX}{OX}$ in fig. 8. If we proceed from the axis to the surface of the wire we get a family of ellipses in which H_0 varies from zero at the axis to $\dfrac{2\sqrt{2}I}{a}$ at the surface, where *a* is the radius and I the R.M.S. value of current. To obtain μ_r in terms of the measured permeability of the wire it is assumed that the locus of P is the B-H curve of the wire, so that the value of μ_r for any value of H_0 is equal to the measured permeability for a field-strength $H = H_0$.

Now except when the frequency is infinitesimally small the variation of H_0 from axis to surface is not linear, so that it is next necessary to determine the distribution of field over the cross-section. It may easily be shown from the classical equation of conduction in a wire that the amplitude of field (H_0) at a point distant r from the axis is given by H_r, where

$$H_r = \frac{(ber'^2\, x + bei'^2\, x)^{\frac{1}{2}}}{(ber'^2\, z + bei'^2\, z)^{\frac{1}{2}}} \cdot H_a \tag{11}$$

in which

$$z = 2\sqrt{\frac{\mu\,\omega l}{R_0}}$$

$$x = \frac{r}{a} \cdot z$$

$$H_a = \frac{2\sqrt{2}I}{a}.$$

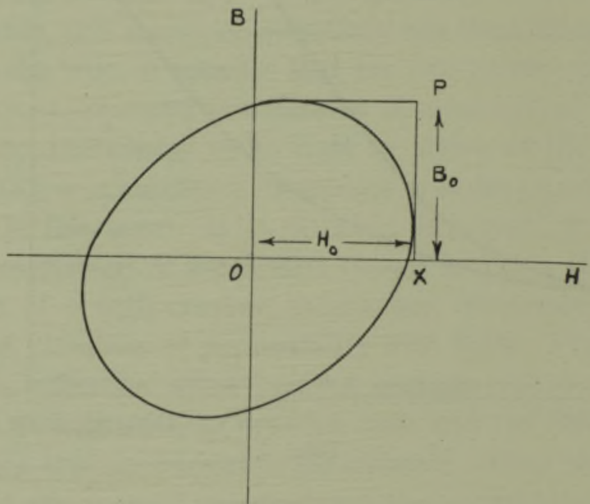

FIG. 8.

Since, however, we have substituted for μ in the classical equation the complex quantity $\mu_1(1 - jb)$ (*vide* p. 465), the parameter z becomes complex, and it is not easy to obtain numerical values of H_r from (11). To a first approximation, therefore, the distribution of field is calculated on the assumption that hysteresis may be ignored, so that $b = 0$, the values of z being obtained by using those of μ given in Table III, *a*. The values of μ_r over the cross-section then follow, according to assumption, from the B-H curve obtained ballistically, and may be plotted to give curves of the type shown in fig. 7.

It is finally necessary to average μ_r over the cross-section in order to establish the value of μ for the whole wire, which should, if the assumptions made are reasonable, be in agreement with that deduced from the impedance measurements and given in Table III, *a*. It is very doubtful how this average should be calculated, but for simplicity it has been taken as the arithmetic mean of μ_r over the cross-section, *i.e.*, we write

$$\mu = \frac{1}{a}\int_0^a \mu_r \, dr.$$

On this basis it has been attempted to construct Table III, *a*, from the B-H curve of the wire, but the results are not particularly encouraging. The calculated maximum values of μ at each frequency are always greater than those shown in the table, and the discrepancy increases with the frequency. Thus, if the distribution of field over the cross-section is linear, which is so for very low frequencies, the calculated value of μ is 6×10^4, that obtained by extrapolation from Table III, *a*, is $5 \cdot 8 \times 10^4$, which is in good agreement. But at 500 cycles per second the experimental value of μ_{max} is $2 \cdot 5 \times 10^4$ while that calculated is about 5×10^4. The theory, therefore, is inadequate to account quantitatively for the decrease in μ_{max} with frequency shown by the impedance measurements. This may indicate that the concentration of current in the surface of the wire is greater than has been calculated approximately on the classical theory. A further discrepancy is that at any frequency the maximum value of μ shown in Table III, *a*, occurs at a value of current considerably greater than that which the theory would indicate. From this it would appear that the amplitude of field at the surface of the wire must actually be less than $\frac{2\sqrt{2}I}{a}$. But there remains the possibility that the hysteresis loop traversed at a point in the wire during a cycle of current cannot be related, as has been assumed, to the loop measured ballistically with longitudinal magnetization, and in this case the whole basis of comparison must break down.*

If we turn to Table III, *b*, and consider the variations in K, the hysteresis constant deduced from impedance measurements, this view appears to find some support, for these variations are very difficult to account for by means of the ballistic hysteresis loops of the material, since we have found that K determined ballistically is appreciably constant over a wide range of flux density. In Table III, *b*, the variations of K with current at constant frequency are perhaps small and irregular enough to be attributed to fortuitous divergences from a constant value which, as has been seen, agrees well enough with the ballistic value when the frequency is sufficiently low. But the marked increase in K which occurs with increasing frequency

* There actually appears to be some evidence of magnetic anisotropy in ferromagnetic wires, *vide* Ermolaev, ' Arch. Elecktrotech.,' vol. 23, p. 101 (1929).

seems to indicate either that the hysteresis loop in these conditions is quite different from that obtained ballistically, or that the postulated lag between field and induction in the wire is due not only to hysteresis but to some other property, such as magnetic viscosity, which increases with frequency.

ii—*The Effect of Applying an Axial Field*—The values of μ and K have been computed as above for the same wire with the same range of frequency and current, and with applied axial fields up to 0·6 gauss. As an example of the effect of axial field the values at a frequency of 55 cycles per second are shown in Table VI, *a* and *b*. The results at higher frequencies are similar in character.

TABLE VI

a—VALUES OF $\mu \times 10^{-4}$. FREQUENCY 55 C./SEC.

Current m/A.	Axial field in gauss 0	0·1	0·2	0·4	0·6
3·0	2·09	1·78	1·32	1·14	1·10
5·0	3·02	2·90	2·10	1·56	1·30
7·5	4·18	3·87	2·69	1·93	1·51
10·0	4·85	4·16	2·93	1·95	1·58
15·0	4·24	3·60	2·72	1·85	1·50
20·0	3·30	2·95	2·33	1·71	1·32
30·0	2·35	2·12	1·86	1·49	1·08
35·0	2·07	1·92	1·68	1·38	1·01

b—VALUES OF $K \times 10^{6}$. FREQUENCY 55 C./SEC.

Current m/A.	0	0·1	0·2	0·4	0·6
3·0	4·7	5·0	5·6	7·0	6·3
5·0	5·0	4·7	5·1	7·2	8·1
7·5	4·2	4·3	4·8	6·3	7·9
10·0	3·4	3·8	4·5	6·4	7·3
15·0	3·4	3·7	4·0	5·5	6·5
20·0	3·9	3·8	4·1	4·4	6·5
30·0	4·0	4·0	4·2	4·7	6·3
35·0	4·3	3·9	4·2	4·5	5·7

Now when a wire is subjected simultaneously to an alternating circular and a direct longitudinal magnetic field, its resultant magnetization is of an alternating screw type, the mathematical investigation of which is not within the scope of this paper. No quantitative explanation of the experimental results shown in Table VI will therefore be attempted. The results can, however, at any rate as far as μ is concerned, be broadly explained by means of the theory of ferro-magnetism. A ferro-magnetic body is conceived to consist of a very large number of domains, which, in the absence of an external field, are spontaneously magnetized in directions distributed at random over the aggregate, so that the resultant magnetization of the body is zero. If the body is a wire and a circular field is applied to it by the passage of current, direct or alternating, the directions of magnetization of the domains tend to conform to the direction of the field and the body becomes circularly magnetized, the circular permeability at any point in the wire assuming the value appropriate to the field at that point. If a direct longitudinal field of increasing magnitude is superposed on an alternating circular field, the directions of magnetization of the domains tend to lie along the axis of the wire, and the constraint thus imposed renders them less able to conform to the alternations of the circular field, and the circular permeability is decreased, slightly for small axial fields, and then at a rate which increases rapidly with the field for a time but finally dies away as the magnetization of the domains approaches perfect alignment with the field and the wire becomes saturated. The values of μ given in Table VI, *a*, show just this type of variation as the axial field is increased.

A somewhat analogous phenomenon is the well-known fact that when a direct current is passed through a wire so as to give it a steady circular magnetization the longitudinal permeability is decreased. On the other hand, when the circular field is alternating the longitudinal permeability for small fields is known to be increased, probably in the same way as it is increased by the agitation due to mechanical vibration. Thus in the case considered here the decrease in circular permeability caused by the axial field is accompanied by an increase in longitudinal permeability.

It will be remembered that when an external field is applied at right angles to the axis of the wire there is no measurable change of impedance, and consequently of mean circular permeability. During any half-cycle of current the applied transverse field and the circular field due to the current are in the same direction in one half of the cross-section of the wire and in opposite directions in the other half. It is to be expected, then, that the effect of a transverse field over the whole cross-section will be zero.

VI—Discussion of Results

The experimental results which have been described in § III show that the A.C. resistance and reactance of a highly permeable wire undergo changes of considerable magnitude when its circular or longitudinal magnetizations are altered by variations in current or external field. It is evident that this variation of impedance with external field affords a novel method of measuring or detecting small changes in a magnetic field of the order of that of the earth, or even of measuring absolute values of field. It is not the purpose of this paper to discuss or describe in any detail such possible applications of the effect which has been discovered, but certain considerations which arise from the experimental results may briefly be touched upon.

We have seen that the reactance of a wire is small compared with its resistance, so that in general the power factor considerably exceeds 0·9. Therefore, although a change in external field produces a change of impedance in the wire, this may be considered for practical purposes to be due to a change of ohmic resistance, the reactive change being negligible. It is desirable at this stage to introduce some quantity which will define the sensitivity of a wire to changes in external field. If in an axial field H the A.C. resistance is R, and if a small change of field δH gives rise to a change of resistance δR, then $\delta R = \frac{dR}{dH} \cdot \delta H$, where $\frac{dR}{dH}$ is the slope of the R-H curve at the point H. We define the sensitivity of the wire in a field H as the percentage change of resistance caused by 0·1 gauss change of field, *i.e.*,

$$S = \frac{100}{R} \cdot \frac{dR}{dH} \times 0\cdot 1 = \frac{10}{R}\frac{dR}{dH}.$$

It will be realized that this is not necessarily in all cases the most suitable definition of sensitivity, but it has been found a convenient term to use in discussing the characteristics of a particular specimen of material.

It is clear at once from the data which have been given that the sensitivity of a wire of given diameter varies between very wide limits according to its length, the external magnetic field, and the amplitude and frequency of the current.

Reference to Table I shows that, *ceteris paribus*, the sensitivity is greatest with the optimum current flowing in the wire. This, it will be recalled, is the current at which the resistance has its maximum value and is therefore least affected by accidental current variations, so that stability as well as sensitivity is a maximum. On the other hand, the sensitivity of any network in which the wire may be placed is proportional to the current, so that in some cases it may be practically advantageous to sacrifice some intrinsic sensitivity and stability in order to obtain greater network sensitivity. Theoretically, however, it is desirable to work at optimum current, and this will be taken as the basis of further discussion.

The R-H curves at different frequencies (using the optimum current) for a particular wire sample are shown in fig. 4. Although the maximum slope of the curve increases with frequency, the sensitivity as defined above in a field of $0 \cdot 4$ gauss (that of the earth's vertical component) is greatest at a frequency of about 500 cycles per second, and this would therefore appear to be in general the best working frequency.

As regards the dimensions of the wire, the curves of fig. 6 show that for a given diameter the sensitivity increases with the length, since with wires of small dimension ratio the effect of self-demagnetization is to reduce the sensitivity in small fields. If the length is less than 20 cm. there is practically no change of resistance in a wire of $0 \cdot 445$ mm. diameter for fields between zero and $0 \cdot 2$ gauss. Adequate length is thus essential. If length is restricted, for any reason, greater sensitivity in small fields may be obtained by using wires of smaller diameter. This, however, introduces considerations other than that of the dimension ratio, since the " skin effect ", which determines the A.C. resistance, is a function of the diameter, and, as we have seen (Table II), wires of different diameters show fundamental differences in behaviour when used as conductors of A.C. Thus the optimum current decreases with the diameter, and the resistance-maximum which occurs at this current becomes sharper the finer the wire, so that a decrease in diameter entails a loss not only of network sensitivity but of stability to accidental current variations.

It is clear that in applying the field-sensitivity of the A.C. resistance of nickel-iron alloy wires to the measurement of magnetic fields or changes of field there exists a wide choice of working conditions, and that the optimum conditions can only be determined by reference to the particular circumstances of the measurement required. Enough has been said to indicate the application to such determination of the experimental results which form the subject matter of this paper.

In the next section some account is given of experiments which have been carried out with the object of producing wires of as high sensitivity as possible by means of suitable heat treatment.

VII—Heat Treatment

It is evident from the theory which has been developed earlier that the change of impedance with axial field is an extremely complex phenomenon, and uncertainty must exist as to the precise magnetic qualities which it is necessary to develop in a wire by heat treatment in order that it should have the greatest possible value of $\frac{1}{R} \cdot \frac{dR}{dH}$. But high permeability in small fields, say less than $0 \cdot 5$ gauss, is clearly essential, and the methods of heat treatment employed have been those found most effective in producing this property. The actual value of the permeability developed in a given wire depends in detail not only on the heat treatment, which must be found by experiment for each particular " wire draw " made from an ingot, but also on the strain history and annealing history during the wire drawing and its antecedent physical processes.

It is not proposed here to discuss in detail the effects either of composition or of strain history; but it is necessary to emphasize the great importance of preventing oxidation. In the present experiments, owing to the small diameter of the specimens, the question of oxidation becomes particularly important, and its prevention imperative. Various ways of avoiding oxidation have been tried, and the results combine to show that heat treatment should be carried out in an atmosphere of hydrogen, either under high pressure or at atmospheric pressure. In order to develop the highest permeability, it was found to be definitely advantageous to take precautions against oxidation, not only during the main heat treatment but during the wire drawing process itself, by performing the intermediate annealings in hydrogen during drawing. As regards the final heat treatments, the heating of the drawn wires for a considerable length of time in hydrogen does more, of course, than merely prevent oxidation: it completes the reduction of any oxide still remaining included in the wire after the drawing.

Although many wires showing unusually high values of mean permeability over ranges of field from 0·4 to 0·5 gauss have been obtained after heating in hydrogen under pressures of 60 lb. per sq. in. (above atmospheric pressure), the present experiments give no direct evidence either for or against the theory proposed by Cioffi* that hydrogen, mechanically retained in the cooled metal, is itself a contributory cause of exceptionally high initial permeability.

The method of experiment eventually developed for finding the best heat treatment for samples taken from a batch of wires drawn from one particular ingot consists in varying separately the maximum temperature to which the sample is heated and the length of time it is held at that temperature. Additional variation is caused by a second or even a third heat treatment to the top temperature. The rate of cooling was sometimes controlled and sometimes that of the furnace itself—but in all cases kept the same when the other variables were altered.

For the binary alloys—78·5% Ni. 21·5% Fe (the original permalloy) the heat treatment recommended by Arnold and Elman† has been found

* Cioffi, ' Nature,' vol. 126, p. 200 (9 August, 1930).

† Arnold and Elman, ' J. Franklin Instn.,' vol. 195, p. 621 (1923).

to be the best. The treatment most likely to be successful with alloys containing copper and chromium, " Mumetal " for instance, is to heat to 1200° C. for several hours and cool slowly with the furnace. It has often been found that a second heating exactly repeating the first materially improved the permeability value of a wire.

The general conclusions reached, as to heat treatment of fine wires of the more complex alloys in which copper and chromium are present, are (i) that special precautions against oxidation must be taken during the wire drawing process as well as during the main heating and that the latter must be done in hydrogen, not only to prevent fresh oxidation but to reduce the oxide already in the wire; (ii) that, in general, high permeability is favoured by the highest annealing temperatures and slow cooling. The effect of short or long exposure to the top temperature remains in doubt.

It has become plain from a large series of measurements of μ in fields of 0·2 gauss made on wires prepared in the way described that a specified heat-treatment applied to individual wires of batch can never be guaranteed to produce the highest value of μ. The results as regards success are analogous to those observed in metal crystal growing: the element of chance plays an important part.

When we proceed to examine the " sensitivity ", as defined on p. 474, of wires in which the appropriate heat treatment has produced high values of permeability, we find a further uncertainty. Although there is a fairly strong correlation between the permeability and the sensitivity of a wire over a given range of fields, we have found that their inter-dependence is far from complete, as is shown by the following experiments.

From several batches of wires, each of 26 S.W.G. and 23 cm. long, of the same composition and subjected to various heat treatments, fifteen wires were selected by a rough ballistic test to fall into a graded series as regards their permeability in a field of 0·2 gauss. The sensitivity $\mathrm{S} = \frac{10}{\mathrm{R}} \cdot \frac{d\mathrm{R}}{d\mathrm{H}}$ of each wire was then measured between 0·2 and 0·3 gauss (where the R-H curve is steepest for wires of this dimension ratio), and the B-H curve of each wire determined ballistically over the range 0–0·5 gauss. From the latter the values of μ in any field were deduced, being defined as the ratio B/H, where H is the external field, and denoting, therefore, the effective permeability in a wire of given dimension ratio. The values of S and those of μ in fields between 0·2 and 0·3 gauss were found to be fairly well correlated, but the relation between the two quantities is far from perfect.

An investigation was therefore made of the frequency distribution of the coefficient S among a large number of wires, all similarly annealed in batches of 50 at a time, and selected so that their μ values fell within a specified narrow range.

The result of two such experiments with separate batches of 340 and 166 wires was to show that the distribution of the values of the coefficient amongst them was governed by the laws of probability. The standard deviation was roughly 10% of the mean value of S.

This is not a very surprising result, since by fixing the permeability of a wire for a single value of field we merely fix one point on its B-H curve. Theory shows that the impedance of the wire depends upon the form of the complete hysteresis loop over a given range of field, so that the frequency distribution of S indicates little more than the possible variations of a hysteresis loop which passes through a single given point. The magnitude of the standard deviation is, however, noteworthy.

To sum up the conclusions as regards heat treatment, we may say that while it is a necessary condition of the production of wires of high S value that they should be so annealed as to be endowed with high permeability in the same field, this condition is not sufficient, and ensures only a certain probability of high S.

In conclusion, the authors wish to express their indebtedness to Mr. S. Butterworth for help and suggestions in the theoretical treatment, and to the Admiralty for permission to publish this paper.

VIII—Summary and Conclusion

The variations of the effective resistance and inductance of nickel-iron wires of high permeability have been investigated when the wires are supplied with alternating current of varying frequency and amplitude. It has also been discovered that large changes in effective resistance occur when external longitudinal fields are applied to the wire. In order to explain the experimental results, the classical theory of alternating-current conduction has been extended to take account of the hysteresis property of ferro-magnetics, but to render the equations soluble, it is necessary to assume a constant value of permeability over the cross-section of the wire, whereas actually the permeability varies from the axis to the circumference. While the experimental results are generally in accordance with theory, it has not been found possible to obtain numerical correlation between the magnetic properties of a wire deduced from the impedance measurements and those measured by direct magnetic methods. Still less is it possible to predict the A.C. resistance or inductance of a wire from its measured magnetic properties.

In a suitably heat-treated wire the change of A.C. resistance caused by the application of small longitudinal fields, of the order of that of the earth, is so large as to suggest the use of the effect in the measurement or detection of small changes of magnetic field. The heat-treatment required to produce the maximum effect depends upon the composition and strain history of the wire, and must be found by experiment. While no detailed general rules can be laid down, annealing in an atmosphere of hydrogen is always an essential. Even when the best treatment for any particular sample has been determined, there is no certainty that it will be effective in producing a high resistance-field sensitivity in any individual wire, for the sensitivity coefficients of a large number of similar wires, similarly annealed, are found to be distributed over a wide range of values, approximately according to the probability law.