Experimental test of the compatibility of the definitions of the electromagnetic energy density and the Poynting vector

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Abstract

It is shown that the generally accepted definition of the Poynting vector and the energy flux vector defined by means of the energy density of the electromagnetic field (Umov vector) lead to the prediction of the different results touching electromagnetic energy flux. The experiment shows that within the framework of the mentioned generally accepted definitions the Poynting vector adequately describes the electromagnetic energy flux unlike the Umov vector. Therefore one can conclude that a generally accepted definitions of the electromagnetic energy density and the Poynting vector, in general, are not always compatible.

keywords: Poynting vector, energy density

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I. INTRODUCTION

In the article "Motion equations of the energy in the bodies" [1] that appeared in the year that "Tractate" was published by Maxwell (1873), Umov developed the consequences from the idea of the energy localization in the mediums. To each volume element in the medium, the particles of which are in movement, an energy, constituted by the sum of the alive forces of the particles and elements and the potential energy, is associated. Umov thinks about the problem of settling down in general form "the laws of the transition of the energy from an element to another", and to determinate, starting from general principles, the study of the movement of the energy in the mediums. Starting from the energy conservation law Umov deduces the motion equation for the energy in the mediums. If we represent the energy density in a given point of the medium by means of w, and through v_x, v_y and v_z the energy velocity components in this point, then the energy density loss in that point in unit of time is determined by the relationship

$$-\frac{\partial w}{\partial t} = \frac{\partial \left(w v_x \right)}{\partial x} + \frac{\partial \left(w v_y \right)}{\partial y} + \frac{\partial \left(w v_z \right)}{\partial z}. \tag{1}$$

> "The expression (1), similar to the expression of the mass conservation law in hydrodynamics, is the expression of the elementary energy conservation law in the mediums", Umov writes. From this expression it can be established "the relationship among the quantity of energy, that in unit time leaves toward the medium through its frontier, and the change of the quantity of energy in the medium".

This relationship is expressed with the integral expression (Umov theorem)

$$\iiint \frac{\partial w}{\partial t} dxdydz + \iint wv_n d\sigma = 0. \qquad (2)$$

The vector $w\mathbf{v}$ defines the energy flow which crosses, in the unit time, the perpendicular to this vector unitary surface. This is the so-called Umov vector.

The case of the electromagnetic field, as particular case of the Umov theorem, and therefore of the Umov vector, was studied by Poynting.

In the year 1884 J. Poynting published the article [2] that contained the previously mentioned Umov-Poynting theorem. In this work Poynting independently arrives to the same point of view developed 10 years before by Umov. Poynting writes:

> "If we recognize the continuity of the energy movement, that is to say we recognize that when the energy disappears in some point and appears in other, it should pass through the intermediate space, then we are obliged to reach the conclusion that the surrounding medium contains at the least a portion of the energy and that it is capable to transmit the energy from one point to another."

Further on Poynting, leaning on the Maxwell idea about the energy localization in the field, formulates in this way the main idea of his work:

> "The objective of this article is to demonstrate that there exists a general law for the energy transport, in agreement with which the energy in any point moves perpendicularly to the plane containing the lines of the electric and magnetic forces, and that the quantity of the energy passing through the unitary surface in this plane, for unit of time, is equal to the product

> of the magnitudes of these two forces multiplied by the sinus of the angle among them and divided among 4π".

By this way Poynting defines the energy-flux vector for the case of the electromagnetic field.

Discussing today the conception of the Poynting vector and the number of basic difficulties associated with this concept one can sense clearly that neither among researchers (see, e.g., [3,4] and corresponding references there) nor among authors of the generally accepted text-books of classical electrodynamics (see, e.g. [5-9]) a general agreement exists about the essence of the energy-flux vector related with electromagnetic fields. Actually, the well-known authors Panofsky and Phillips state [5]:

> "Paradoxical results may be obtained if *one tries to identify* the Poynting vector with the energy flow per unit area at any particular point".

Contrarily, Feynman states [9] that *exclusively* the identification of the Poynting vector (in its generally accepted form) with the energy flow per unit area allows to understand the law of conservation of the angular momentum in some special cases. Other well-known authors Landau and Lifshitz state [6]:

> "Therefore the integral $\oint \mathbf{S} d\mathbf{f}$ must be interpreted as the flux of field energy across the surface bounding the given volume, *so that the Poynting vector* $\mathbf{S}$ *is this flux density* – the amount of field energy passing through unit area of the surface in unit time."

Tamm [7] also identifies the Poynting vector with the energy flow per unit area at any particular point, however, taking into account that the definition

$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{H})$ is not unique. In turn Jackson claims in his famous text-book [8]:

> "The vector $\mathbf{S}$, *representing energy flow*, is called the Poynting vector. It is given by $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ (6.109) [1].... Relativistic considerations (Section 12.10) show that (6.109) is *unique*."

The only way we can verify the standard formula for the energy flow due to the electromagnetic field is by experiment. Feynman said [9]:

> "There are, in fact, an infinite number of possibilities for w (energy density) and $\mathbf{S}$, and so far no one has thought of an experimental way to tell which one is right."

In this work we theoretically rationalize that the Poynting vector (in its standard definition) does not *always* coincide with the energy flux vector (Umov vector) related with electromagnetic waves. The results of the experiment show that the Poynting vector is not always compatible with the generally accepted definition of the electromagnetic energy density.

II. THEORETICAL MOTIVATION OF THE EXPERIMENT

More often than not physicists implicitly suppose that the Poynting vector

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B} \tag{3}$$

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[1] Jackson uses SI

and Umov (energy flux) vector[2]

$$\mathbf{U} = wv\mathbf{n} \tag{4}$$

always coincide for *any* electromagnetic wave spreading in vacuum in every point. Here $\mathbf{n}$ is a unit vector along the direction of propagation of the electromagnetic energy, v is the transferring energy velocity (in the case of electromagnetic waves in vacuum $v = c$) and w is the energy density of the electromagnetic wave. In actual fact, this assertion is shown at least for plane and spherical electromagnetic waves in vacuum (see, e.g., [6], Eq. 47.5). Nevertheless, the assertion that $\mathbf{S} = \mathbf{U}$ for waves of a more general kind is not proved in textbooks and monographs.

Let us study what condition in vacuum for $\mathbf{E}$ and $\mathbf{B}$ in an electromagnetic wave must be satisfied when the equality $\mathbf{S} = \mathbf{U}$ is valid. We have in CGS (Gauss' system):

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E} \times \mathbf{B} = \frac{c}{4\pi}EB\sin\alpha\mathbf{n} \tag{5}$$

and

$$\mathbf{U} = wc\mathbf{n} = \frac{c}{8\pi}(E^2 + B^2)\mathbf{n}. \tag{6}$$

Equating (5) and (6) we obtain

$$2EB\sin\alpha = E^2 + B^2 \tag{7}$$

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[2]The expression of the Umov vector ($\mathbf{U} = w\mathbf{v}$) is obtained from the general energy conservation law ($\frac{\partial w}{\partial t} = -\nabla\{w\mathbf{v}\}$) and describes the energy flux density of *any* kind of energy (not only electromagnetic energy), w is the corresponding energy density and $\mathbf{v}$ is the propagation velocity of the energy in a given point. Thus the Poynting and Umov vectors should always coincide.

or

$$(E - B)^2 + 2EB(1 - \sin\alpha) = 0. \tag{8}$$

According to the problem definition we choose real values of E, B and α only, where α is the angle between $\mathbf{E}$ and $\mathbf{B}$. Therefore the last equality (8) can be valid if and only if $E = B$ and $\alpha = \pi/2$. Thus the **Theorem** takes place: *for the equality of the Poynting vector and Umov vector it is necessary and sufficient that* $\mathbf{E}\perp\mathbf{B}$ *and* $E = B$.

In the next sections we propose and perform the experiment which allows us to check the incompatibility of the conventional functional forms of the Poynting vector and the electromagnetic energy density when the electromagnetic wave field does not satisfy the conditions

$$\mathbf{E}\perp\mathbf{B} \qquad \text{and} \qquad E = B. \tag{9}$$

III. THEORETICAL PREDICTIONS

In order to obtain theoretically the electromagnetic energy flow observed in the experiment described in section IV, we model the sources with the help of two point sources emitting spherical waves and we calculate the flow intensity by using the definition of the electromagnetic energy flow according to the Poynting vector and according to the Umov one.

Let the source 1 be placed in $(0, -l, 0)$ and the source 2 in $(0, l, 0)$. The screen is placed in the plane $z = h$ with $-a \leq x \leq a$ and $-b \leq y \leq b$ (Fig. 1.)

FIGURES

FIG. 1. Position of the point sources and the screen.

The monochromatic spherical waves created by these sources can be modeled with the following expressions for the electric and magnetic fields:

$$\mathbf{E}_1 = \frac{E_0}{R_1} \cos\left(\mathbf{k}_1 \cdot \mathbf{R}_1 - \omega t\right) \mathbf{e}_{\theta 1}, \tag{10}$$

$$\mathbf{B}_1 = \frac{B_0}{R_1} \cos\left(\mathbf{k}_1 \cdot \mathbf{R}_1 - \omega t\right) \mathbf{e}_{\varphi 1} \tag{11}$$

$$\mathbf{E}_2 = \frac{E_0}{R_2} \cos\left(\mathbf{k}_2 \cdot \mathbf{R}_2 - \omega t\right) \mathbf{e}_{\theta 2}, \tag{12}$$

$$\mathbf{B}_2 = \frac{B_0}{R_2} \cos\left(\mathbf{k}_2 \cdot \mathbf{R}_2 - \omega t\right) \mathbf{e}_{\varphi 2}, \tag{13}$$

where $\mathbf{k}_1 = \frac{k}{R_1}\mathbf{R}_1$ y $\mathbf{k}_2 = \frac{k}{R_2}\mathbf{R}_2$, E_0 and B_0 are amplitudes, $k = 2\pi/\lambda$ is wave number for both waves. By this way the energy flow has radial direction for each of these sources

$$\mathbf{S}_1 = \frac{cE_0B_0}{4\pi R_1^2} \cos^2\left(\mathbf{k}_1 \cdot \mathbf{R}_1 - \omega t\right) \mathbf{e}_{R1}, \tag{14}$$

$$\mathbf{S}_2 = \frac{cE_0B_0}{4\pi R_2^2} \cos^2\left(\mathbf{k}_2 \cdot \mathbf{R}_2 - \omega t\right) \mathbf{e}_{R2}, \tag{15}$$

where $\mathbf{e}_{R1}$, $\mathbf{e}_{\theta 1}$, $\mathbf{e}_{\varphi 1}$ and $\mathbf{e}_{R2}$, $\mathbf{e}_{\theta 2}$, $\mathbf{e}_{\varphi 2}$ are the corresponding local spherical orts associated to the sources 1 and 2. By means of r, θ y φ we will designate the spherical coordinates in our coordinate system. The corresponding electromagnetic energy densities are defined by the expressions

$$w_1 = \frac{E_0^2 + B_0^2}{8\pi R_1^2} \cos^2\left(\mathbf{k}_1 \cdot \mathbf{R}_1 - \omega t\right), \tag{16}$$

$$w_2 = \frac{E_0^2 + B_0^2}{8\pi R_2^2} \cos^2 (\mathbf{k}_2 \cdot \mathbf{R}_2 - \omega t) \,. \tag{17}$$

If we take into account that $E_0 = B_0$, then for each of the two spherical waves, created by the sources 1 and 2, the conditions $E = B$ and $\mathbf{E} \perp \mathbf{B}$ are fulfilled and therefore the Poynting and the Umov vectors coincide.

$$\mathbf{S}_1 = \mathbf{U}_1, \tag{18}$$

$$\mathbf{S}_2 = \mathbf{U}_2. \tag{19}$$

Let us consider now the resulting electromagnetic field by these two sources at the same time

$$\mathbf{E}_T = \mathbf{E}_1 + \mathbf{E}_2, \tag{20}$$

$$\mathbf{B}_T = \mathbf{B}_1 + \mathbf{B}_2. \tag{21}$$

In this case the Poynting vector and the Umov vector will have the form

$$\mathbf{S}_T = \frac{c}{4\pi} \mathbf{E}_T \times \mathbf{B}_T = \mathbf{S}_1 + \mathbf{S}_2 + \frac{c}{4\pi} (\mathbf{E}_1 \times \mathbf{B}_2 + \mathbf{E}_2 \times \mathbf{B}_1) \tag{22}$$

and

$$\mathbf{U}_T = w_T c \mathbf{n}, \tag{23}$$

where

$$w_T = \frac{E_T^2 + B_T^2}{8\pi} = w_1 + w_2 + \frac{1}{4\pi} (\mathbf{E}_1 \cdot \mathbf{E}_2 + \mathbf{B}_1 \cdot \mathbf{B}_2) \tag{24}$$

and where $\mathbf{n}$ is the direction of the electromagnetic energy propagation for the resulting field, that is to say the unitary vector in the direction of the vector $\mathbf{S}_T$.

The energy flow measured experimentally is the integral over the screen surface (with normal $\mathbf{k}$, unitary vector in the positive direction of the Z axis) of the temporary average of the energy flux density. For each source we have

$$\Phi_1 = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{S}_1 \cdot \mathbf{k} \rangle_t \, dxdy = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{U}_1 \cdot \mathbf{k} \rangle_t \, dxdy, \tag{25}$$

$$\Phi_2 = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{S}_2 \cdot \mathbf{k} \rangle_t \, dxdy = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{U}_2 \cdot \mathbf{k} \rangle_t \, dxdy, \tag{26}$$

and for the resulting field according the Poynting definition

$$\Phi_P = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{S}_T \cdot \mathbf{k} \rangle_t \, dxdy \tag{27}$$

and according Umov definition

$$\Phi_U = \int_{-b}^{b}\int_{-a}^{a} \langle \mathbf{U}_T \cdot \mathbf{k} \rangle_t \, dxdy, \tag{28}$$

where the designation $\langle \ldots \rangle_t$ is the time average value.

Laborious calculations show that

$$\Phi_1 = \Phi_2 = \frac{chE_0B_0}{8\pi}\int\int \frac{dxdy}{R_1^3} = \frac{chE_0B_0}{8\pi}\int\int \frac{dxdy}{R_2^3} \equiv \Phi_0 \tag{29}$$

and that the exact relationships between the resulting flow and the flows for separate sources ($\mathcal{K} = \frac{\Phi}{2\Phi_0}$) are

$$\mathcal{K}_P = \frac{\Phi_P}{2\Phi_0} = 1 + \frac{\gamma}{2\beta} \tag{30}$$

for the Poynting definition, and for the Umov definition

$$\mathcal{K}_U = \frac{\Phi_U}{2\Phi_0} = \frac{\kappa}{\beta}, \tag{31}$$

where

$$\gamma = \int_{-b}^{b}\int_{-a}^{a} \frac{\cos\left(k\left(R_1 - R_2\right)\right)}{U_1 U_2 R_1^2 R_2^2} \left[\frac{\left(R_1 + R_2\right) R_2 R_1}{r^2}\sin^2\theta \right.$$
$$-2\frac{l}{r}\left(R_2 - R_1\right)\sin^3\theta\sin\varphi$$
$$\left. -\left(\frac{l}{r}\right)^2 \left(R_1 + R_2\right)\left(2\sin^2\theta\sin^2\varphi + \cos^2\theta + 2\sin^3\theta\cos^2\varphi\right)\right] dxdy, \quad (32)$$

$$\beta = \int_{-b}^{b}\int_{-a}^{a} \frac{dxdy}{R_1^3}, \quad (33)$$

$$\kappa = \int_{-b}^{b}\int_{-a}^{a}\int_{0}^{\frac{2\pi}{kc}} \frac{\left(F + GH\right)\left(J + KL\right)}{\left(A + B + C + 2D + 2E\right)^{1/2}} dtdxdy, \quad (34)$$

$$A = \frac{1}{R_1^4}\cos^4\left(kR_1 - \omega t\right), \quad (35)$$

$$B = \frac{1}{R_2^4}\cos^4\left(kR_2 - \omega t\right), \quad (36)$$

$$C = \left(\frac{r}{U_1 U_2 R_1^2 R_2^2}\right)^2 \cos^2\left(kR_1 - \omega t\right)\cos^2\left(kR_2 - \omega t\right)$$
$$\times\left\{\left[\left(R_1 + R_2\right)\frac{R_2 R_1}{r^2}\sin^2\theta + \left(R_1 U_2^2 - R_2 U_1^2\right)\frac{l}{r}\sin\theta\sin\varphi - \left(R_2 + R_1\right)\left(\frac{l}{r}\right)^2\cos^2\theta\right]^2 + \right.$$
$$\cos^2\theta\left[\left(R_2 - R_1\right)\sin^2\theta\sin\varphi + 2\frac{l}{r}\left(R_2 + R_1\right)\sin\theta\cos^2\varphi - \left(\frac{l}{r}\right)^2\left(R_2 - R_1\right)\sin\varphi\right]^2 +$$
$$\left.\cos^2\varphi\left[\frac{R_1 R_2}{r^2}\left(R_2 - R_1\right)\sin^2\theta - 2\frac{l}{r}\left(R_1 + R_2\right)\cos^2\theta\sin\theta\sin\varphi + \left(\frac{l}{r}\right)^2\left(R_1 - R_2\right)\cos^2\theta\right]^2\right\}, \quad (37)$$

$$D = \frac{r^2}{R_1^3 R_2^3}\left[1 - \left(\frac{l}{r}\right)^2\right]\cos^2\left(kR_1 - \omega t\right)\cos^2\left(kR_2 - \omega t\right) \quad (38)$$

$$E = \frac{r^2}{U_1 U_2 R_1^2 R_2^2} \cos(kR_1 - \omega t) \cos(kR_2 - \omega t) \times \left[\frac{1}{R_1^2} \cos^2(kR_1 - \omega t) + \frac{1}{R_2^2} \cos^2(kR_2 - \omega t) \right]$$
$$\times \left[\left(1 + \frac{R_1 R_2}{r^2}\right) \sin^2\theta - \left(\frac{l}{r}\right)^2 \left(1 + \frac{R_1 R_2}{r^2} - 3\sin^2\theta + 4\sin^2\theta \sin^2\varphi\right) + \left(\frac{l}{r}\right)^4 \right] \quad (39)$$

$$F = \frac{1}{R_1^3} \cos^2(kR_1 - \omega t) + \frac{1}{R_2^3} \cos^2(kR_2 - \omega t), \quad (40)$$

$$G = \frac{1}{U_1 U_2 R_1^2 R_2^2} \cos(kR_1 - \omega t) \cos(kR_2 - \omega t), \quad (41)$$

$$H = (R_1 + R_2) \frac{R_2 R_1}{r^2} \sin^2\theta - 2(R_2 - R_1) \frac{l}{r} \sin^3\theta \sin\varphi$$
$$- \left(\frac{l}{r}\right)^2 (R_1 + R_2) \left[2\sin^2\theta \sin^2\varphi + \cos^2\theta + 2\sin^3\theta \cos^2\varphi\right], \quad (42)$$

$$J = \frac{1}{R_1^2} \cos^2(kR_1 - \omega t) + \frac{1}{R_2^2} \cos^2(kR_2 - \omega t), \quad (43)$$

$$K = \frac{r^2}{U_1 U_2 R_1^2 R_2^2} \cos(kR_1 - \omega t) \cos(kR_2 - \omega t), \quad (44)$$

$$L = \left(1 + \frac{R_1 R_2}{r^2}\right) \sin^2\theta - \left(\frac{l}{r}\right)^2 \left(1 + \frac{R_1 R_2}{r^2} - 3\sin^2\theta + 4\sin^2\theta \sin^2\varphi\right) + \left(\frac{l}{r}\right)^4, \quad (45)$$

$$U_1 = \left(\sin^2\theta + 2\frac{l}{r} \sin\theta \sin\varphi + \frac{l^2}{r^2}\right)^{\frac{1}{2}}, \quad (46)$$

$$U_2 = \left(\sin^2\theta - 2\frac{l}{r} \sin\theta \sin\varphi + \frac{l^2}{r^2}\right)^{\frac{1}{2}}, \quad (47)$$

$$R_1^2 = r^2 + l^2 + 2lr\sin\theta\sin\varphi, \tag{48}$$

$$R_2^2 = r^2 + l^2 - 2lr\sin\theta\sin\varphi. \tag{49}$$

The $\mathcal{K}_P$ and $\mathcal{K}_U$ values as a function of the angle α between the rays, were obtained numerically for the values $\sqrt{l^2+h^2} = 0.3\,\mathtt{m}$, $\lambda = 632.8\,\mathtt{nm}$, $a = 3.5\,\mathtt{mm}$ and $b = 2.5\,\mathtt{mm}$ used in the experiment. The graph of these dependencies are presented in the Fig. 2.

FIG. 2. Theoretical curves of the behavior of the coeficient $\mathcal{K}$ versus angle according the Poynting and Umov vectors.

It is not difficult to show that the previous results for the relationship between the resulting flow and the flows for separate sources ($\mathcal{K} = \frac{\Phi}{2\Phi_0}$) are conserved if the spherical waves 1 and 2 are modeled by means of the equations

$$\mathbf{E}_1 = \frac{E_0}{R_1}\cos\left(\mathbf{k}_1\cdot\mathbf{R}_1 - \omega t\right)\mathbf{e}_{\varphi 1}, \tag{50}$$

$$\mathbf{B}_1 = -\frac{B_0}{R_1}\cos\left(\mathbf{k}_1\cdot\mathbf{R}_1 - \omega t\right)\mathbf{e}_{\theta 1}, \tag{51}$$

$$\mathbf{E}_2 = \frac{E_0}{R_2}\cos\left(\mathbf{k}_2\cdot\mathbf{R}_2 - \omega t\right)\mathbf{e}_{\varphi 2}, \tag{52}$$

$$\mathbf{B}_2 = -\frac{B_0}{R_2}\cos\left(\mathbf{k}_2\cdot\mathbf{R}_2 - \omega t\right)\mathbf{e}_{\theta 2}. \tag{53}$$

IV. DESCRIPTION OF THE EXPERIMENT

The diagram of the experimental arrangement is shown in the Fig.3

FIG. 3. The experimental arrangement diagram. I1 and I2 are optical obturators. ADC is the analog digital converter.

As coherent monochrome light source a He-Ne laser model 08181.93 from the company "PHYWE" with parameters: wavelength $l = 632.8$nm, beam power $P = 1$mW, polarization $500 : 1$, beam diameter $d = 0.5$mm, is used. The laser beam goes to the beamsplitter $B1$, where it is unfolded in two rays of the same intensity approximately. The reflected ray goes to the reference photodiode PIN1, model 1PP75 from the company "TESLA", that works in short circuit regime. The photodiode-generated current is amplified by an amplifier DC and by means of an analogical-digital converter ADC is received on the computer PC. This signal serves as a reference signal and gives us information about the intensity changes of the laser beam.

The part of the laser beam, which goes through the beamsplitter B1, goes to the second beamsplitter B2, where on the other hand it is unfolded in two rays. The second-beamsplitter-reflected ray goes to the optic obturator I2, then reflected by the mirror M2 and falls on the lens L2. The lens L2 is a biconvex lens and possesses focal distance of 18mm. This lens transforms the cylindrical ray in a divergent beam. After going through the lens the beam goes to the measuring photodiode PIN2 (type 1PP75 from "TESLA") and it falls on its active surface under an angle α. The ray that goes through the beamsplitter B2, goes consecutively through the optic obturator I1 and the adjustable compensator C, then is reflected by the mirror M1 and falls on the lens L1. The lens L1 is of the same type as the lens L2 and has the same function. After going through the lens L1 the divergent beam, produced by the lens L1, falls on the measuring photodiode under the same angle. The angle between both beams falling on the PIN2 is equal to 2α (Fig.3). The photodiode PIN2 also works in short circuit regime. Its signal is amplified by another amplifier DC and goes through an analogical-digital converter toward the computer. Its signal is proportional to the luminous flow that falls on its active surface.

The mirrors M1 and M2 are movable. The distances between each lenses and the measuring photodiode PIN2 are the same and they measure 30cm. These distances stay fixed during the experiment. The angle α is changed only from 14^o until 86^o and its value is measured within the accuracy of 0.5^o. When carrying out the experiment the result of each measurement is corrected with the reading of the photodiode PIN1. By this way the error produced by the laser instability is avoided.

All the measurements are carried out in a dark room. To avoid the influence of the laser instability on the experimental results, a normalization of the readings of the photodiode PIN2 is executed with the help of photodiode PIN1 for all the measurements. For each value of the angle α the experiment is executed in three stages:

Stage 1: Both optical obturators I1 and I2 are closed up, and by means of the photodiode PIN2 the ground is measured. As the ground value was always below 0.5% of all other measurements, any correction is not applied to the experimental results.

Stage 2: This stage has as a goal to equal and to measure the light energy flows that go through both optical branches in the experimental arrangement: branch 1 (optical obturator I1, adjustable compensator C, mirror M1, lens $L1$) and branch 2 (optic obturator I2, mirror M2, lens L2). The obturator I1 closes up and the obturator I2 opens up. The photodiode-PIN2-generated current is measured. Then the obturator I1 opens up and the obturator I2 closes up. By means of the adjustable compensator C the present current in the photodiode PIN2 is adjusted similarly to its previous current with an error limited to 1%. By this way the readings of the photodiode PIN2 that correspond to the optical flow Φ_1 that passes through the branch 1, and also correspond to the optical flow Φ_2, that passes through the branch 2, are already known.

Stage 3: Both obturators I1 and I2 open up and the current of the photodiode PIN2 that represents the total flow Φ is measured. Then the computer PC calculates the coefficient

$$\mathcal{K} = \frac{\Phi}{\Phi_1 + \Phi_2}, \tag{54}$$

and it memorizes these values as a function of the angle α.

The experiment was carried out for the two light flows configurations shown in the figs. 4-5,

FIG. 4. Equivalent scheme of the experimental arrangement when the electric field vectors from the wave in the branch 1 are parallel to the electric field vectors from the wave in the branch 2, 2α is the angle between straights connecting each of two sources with the centre of the sensor

FIG. 5. Equivalent scheme of the experimental arrangement when the magnetic field vectors from the wave in the branch 1 are parallel to the magnetic field vectors from the wave in the branch 2.

which correspond to the Eqs. (10)-(13) and (50)-(53). The experimental results are presented in the graphs (fig. 6 and fig. 7).

FIG. 6. The theoretical curves of the behavior of the coeficient $\mathcal{K}$ versus angle according to the Poynting vector and according to the Umov vector and the experimental results for the case of the waves with parallel magnetic field vectors.

FIG. 7. The theoretical curves of the behavior of the coeficient $\mathcal{K}$ versus angle according to the Poynting vector and according to the Umov vector and the experimental results for the case of the waves with parallel electric field vectors.

V. CONCLUSIONS

Our work starts from the position that the Umov vector defines, in a general way, the energy flow for *any* type of energy and it is a consequence of the energy conservation law. Its expression for the particular case of electromagnetic waves is $\mathbf{U} = wc\mathbf{n}$. On the other hand the Poynting vector defines the flow of the electromagnetic energy as a consequence of the energy conservation law in the Maxwell's theory.

However, although both these vectors represent the same physical quantity and therefore they *must* coincide, we demonstrated that their equality is limited to the case when the fields, generating the electromagnetic energy flow, fulfill the conditions $E = B$ (in CGS) and $\mathbf{E} \perp \mathbf{B}$.

Obviously these conditions are not general and therefore one can find situations when the electromagnetic field does not fulfill one or both these conditions. In order to determine which of the definitions of the energy density flux vector (Umov's vector or Poynting's vector), in the case when these conditions are not fulfilled, gives correct prediction about the energy flow, in the present paper we experimentally measured the flow of the resultant electromagnetic energy of two bunches of electromagnetic waves when aforementioned conditions are not fulfilled. This experiment shows that the

Umov vector does not describe appropriately the electromagnetic energy flow, while the Poynting vector *does*.

Therefore, the experiment apparently shows that the energy flux density definition through the Umov vector *in general* is not applicable to electromagnetic phenomena. However *such* conclusion *must be* necessarily incorrect. Indeed, the expression for the Umov vector is obtained starting from the universally accepted conservation law of any type of energy. Consequently the Umov and Poynting vectors should always coincide. For this reason there is an apparent contradiction in the fact that the flow theoretically predicted on the basis of Umov vector coincides neither with the experimentally measured flow nor with the corresponding flow calculated by means of the Poynting vector.

The explanation for this apparent contradiction can reside in the following:

Because the Umov vector is a consequence of the energy conservation law, the Umov vector functional dependence ($\mathbf{U} = wc\mathbf{n}$) should be correct and therefore it is necessary to examine the elements used in its construction, namely, w (energy density), c and $\mathbf{n}$.

The propagation velocity of the electromagnetic waves (and consequently, of the electromagnetic energy) in vacuum is unique and is equal to c, and therefore there is no problem.

However, on the one hand, if the standard definition of the Poynting vector is correct it defines the correct direction of the electromagnetic energy flux and, in turn, obviously, the direction of the Umov vector, then the standard expression of the electromagnetic density utilized to build the Umov vector *cannot be* correct. On the other hand if the standard definition of the energy density is correct then the unit vector $\mathbf{n}$ (direction of the

Poynting vector) used for constructing the Umov vector must be incorrect and therefore the direction of the energy propagation is not correct. This would mean that the standard expression for the Poynting vector does not define correctly the electromagnetic energy density flux. Note, however, that our calculations and experiment in which we used the standard expression of the Poynting vector do not contradict the last claim: the point is that the Poynting vector is defined with the accuracy of the curl of an arbitrary vector and for this reason, in principle, it is possible that the same result for the integral (27) will be obtained from another expression for the electromagnetic flux density. If this is the case then the direction of the energy flow does not have the direction of $\mathbf{E} \times \mathbf{B}$.

So for the case examined by us there is an incompatibility between the generally accepted definition of the electromagnetic energy density and the conventional definition of the energy flux density expressed by the Poynting vector. This particular case allows us to affirm that, in general, these standard definitions *are incompatible*.

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REFERENCES

[1] N. A. Umov, Izbr. soch (selected works)., (Moscow, Gostehizdat, 1950) p. 154. (in Russian).

[2] J. H. Poynting, Phil. Trans, Roy. Soc. London **A175**, 343 (1884).

[3] J. W. Butler, Am. J. Phys. **37**(12), 1258 (1969).

[4] D. E. McLennan, Physics Essays **1**(3), 179 (1988).

[5] W. K. H. Panofsky and M.Phillips, *Classical Electricity and Magnetism* (Addison-Wesley Publ. Co., Reading, Mass., 1962).

[6] L. D. Landau and E. M. Lifshitz, *Teoria Polia* (Nauka, Moscow, 1973) [English translation: *The Classical Theory of Field* (Pergamon, Oxford, 1975)]

[7] I. E. Tamm, *Fundamentals of the Theory of Electricity* (Mir Publishers, Moscow, 1957).

[8] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 3rd ed. 1998).

[9] R. P. Feynman, R. B. Leighton, and M. Sands, *Lectures on Physics* (Addison Wesley, 1965), Vol. 2.