

Vacuum electromagnetic interaction

B R De††

† Department of Electrical and Computer Engineering, University of California, San Diego,
La Jolla, California 92093-0407, USA

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Abstract. The concept of a magnetic 'companion wave' arising when an electromagnetic wave is superimposed on a static magnetic field in vacuum is discussed. A conceptual device for observing vacuum electromagnetic momentum is proposed. The companion wave is then shown to be as real and observable as the electromagnetic wave, and also to have the possibility of carrying information.

1. Introduction

Electromagnetic (EM) interactions occurring in vacuum (e.g. an EM wave impinging on a static magnetic field) are at present thought to be an unobservable and hence inconsequential subtlety of the EM theory [1]. This view, however, is based on the non-existence of a device with which to observe vacuum electromagnetic interactions (hereinafter VEI). In this paper we show that VEI produces real and observable effects such as a heretofore unknown wave behaviour, and present the concept of a device with which to observe such waves. We begin with a discussion of certain developments in EM theory that lead us to take up anew the subject of VEI.

2. Background

The dielectric polarization current J_d , when extrapolated to vacuum, leads to the concept of vacuum displacement current, which is included in the fourth Maxwell equation, written here for a purely dielectric material in MKS units:

$$\nabla \times B = \mu_0 J_d + \mu_0 \epsilon_0 \dot{E} \quad (1)$$

where B and E are the magnetic and the electric fields, and μ_0 and ϵ_0 are the permeability and the permittivity of free space. This vacuum displacement current $J_0 = \epsilon_0 \dot{E}$ leads to electromagnetic waves in free space. A logical complement of this concept arises from a cumulation of evolving ideas to date. Out of a long-standing controversy regarding the form of electromagnetic energy-momentum tensor [2-5], and out of a series of experiments and related further controversy [6-15], at least one simple, clear fact seems now to emerge: a dielectric carrying a displacement current in a magnetic field is subject to a mechanical force $F_d = J_d \times B$ per unit volume. This provides the dielectric counterpart of the $J \times B$ force in a conductor (J = the conduction current). This force has been used, for example,

† Address for correspondence. PO Box 1636, Laguna Beach, California 92652, USA.

to develop the dielectric counterpart of magnetohydrodynamics [16–19]. It is instructive to examine the consequence of extrapolating this dielectric force to vacuum.

By taking the cross product of both sides of equation (1) with B , we have

$$(\nabla \times B) \times B = \mu_0 F_d + \mu_0 \epsilon_0 \dot{E} \times B. \quad (2)$$

This is the 'force equation' corresponding to equation (1) or the 'current equation'. The right-hand side includes a term $F_0 = J_0 \times B$ that has the dimensions of a force, and needs interpretation. We now proceed to show that just as the last term of equation (1) leads to real and observable EM waves in vacuum, the last term of equation (2) leads to real and observable magnetic pressure or energy density waves that accompany the EM waves.

3. An electromagnetic companion wave

We consider a plane EM wave propagating in free space in a region of a homogeneous static magnetic field B_0 parallel to the z direction of a Cartesian coordinate system. The electric and the magnetic field amplitudes of the wave propagating in a direction r may be written as

$$E = E_0 \sin(\omega t - k_0 r) \quad (3)$$

$$b = b_0 \sin(\omega t - k_0 r) \quad (4)$$

where E_0 and b_0 ($= E_0/c$, c = the velocity of light) are the amplitudes, ω is the circular frequency and $k_0 = 2\pi/\lambda_0$ is the propagation constant (λ_0 = the wavelength). The instantaneous net magnetic field is $B = B_0 + b$. The instantaneous energy flow in the medium is given formally by the Poynting vector

$$S = (E \times B)/\mu_0. \quad (5)$$

Unless otherwise specified, we assume in the following discussion a region of space that is so far removed from the sources of the static magnetic field and the EM wave that the time-scale of interaction between the wave and the magnetic field is much shorter than the time needed for these sources to sense this disturbance. Stated differently, we assume that the sources do not contribute to local energy conservation during the interaction.

We next consider successively the following three situations: the wave propagates in the z direction, and the magnetic field b is parallel to the x axis; the wave propagates in the y direction, and the magnetic field b is parallel to the x axis; the wave propagates in the x direction and the magnetic field b is parallel to the z axis. Then the instantaneous Poynting vectors for the above three cases are, respectively,

$$S_z = (c/\mu_0)b_0^2 \sin^2(\omega t - k_0 z) \hat{a}_z \quad (6)$$

$$S_y = (c/\mu_0)b_0^2 \sin^2(\omega t - k_0 y) \hat{a}_y \quad (7)$$

$$S_x = (c/\mu_0)b_0^2 \sin^2(\omega t - k_0 x) \hat{a}_x + (c/\mu_0)B_0 b_0 \sin(\omega t - k_0 x) \hat{a}_x. \quad (8)$$

Thus the wave travelling in the x direction involves a component of energy that travels back and forth. It is an interaction energy in that it involves the wave magnetic field b and

the ambient magnetic field B_0 , and can, in principle, be arbitrarily large compared with the energy flow of the wave itself, i.e. the first term in equation (8).

The Poynting vector sometimes represents a real energy flow (as in the case of EM waves) and sometimes it is only a mathematical term (as when both the electric and the magnetic fields are static). To ascertain which is the case in the present instance, we note that the above result can also be arrived at from first principles without reference to the Poynting vector, from simple work-energy considerations. The total energy density u_{\perp} upon establishing a magnetic field b perpendicular to a pre-existing field B_0 is simply the sum of the energy densities of the two fields:

$$u_{\perp} = (B_0^2 + b^2)/2\mu_0. \quad (9)$$

However, the net energy density when the two fields are parallel or antiparallel is

$$u_{\parallel} = (B_0^2 + b^2 \pm 2B_0b)/2\mu_0. \quad (10)$$

The last term in parenthesis represents the work performed by the wave on the ambient field or vice versa. This is the interaction energy density U :

$$U = B_0b/\mu_0. \quad (11)$$

This term, when multiplied by c , is the same as the last term in equation (8). It represents a spatial and temporal oscillation of the magnetic pressure or magnetic energy density in the medium, the energy being transported back and forth with velocity c , and parallel to the direction of wave propagation. This can also be seen by considering a box enclosing a volume V , with its sides parallel to the coordinate planes making up the surface A . From equations (8) and (10), and leaving out the energy balance for the electromagnetic wave, we can derive the following energy conservation relation:

$$\int \dot{U} dV = \int S_{xc} \cdot \hat{n} dA \quad (12)$$

where S_{xc} is the last term of equation (8), and \hat{n} is the surface normal. The energy transport has certain characteristics of an EM wave in that it represents energy propagating at a velocity c , has a periodicity in time, and involves orthogonal electric and magnetic fields. It is not a modified form or a variant of conventional EM waves, but is something that exists in addition to, and in association with, such waves. In this sense it is a 'companion wave'. While the concept of an EM wave follows from the last term in the current equation (1), that of the companion wave follows from the last term in the force equation (2). Since no energy is being created or absorbed in the medium, the time average of U over one period or its space average over one wavelength must be zero. The companion wave is associated with compressions and rarefactions of the magnetic line of force, much like magnetosonic waves [20].

By applying equation (2) to the case that B_0 is parallel to b , and using the Maxwell's equation $\nabla \times E = -\dot{b}$, we obtain

$$\ddot{U} = c^2 \frac{\partial^2 U}{\partial x^2} \quad (13)$$

which is the wave equation for the companion wave. Or, simply dividing both sides of the above equation by B_0 , we obtain the wave equation for the b field of the EM wave. This shows the nature of the interdependence of the two waves.

4. Observability of vacuum electromagnetic interaction: the force-measuring antenna

Today, EM waves—in particular radio waves—would also be considered inconsequential had it not been for a device with which to observe these waves: an antenna. In the same way, the companion wave becomes consequential when we conceive of a corresponding device: a force-measuring antenna (FMA).

An FMA detects both EM waves and EM momentum. Its concept is simple: it is an antenna mounted in a force-measuring device which is mounted on a rigid body fixed in space (figure 1). Consider for simplicity an FMA made of an ideal short electrical dipole [22] of length L ($\ll \lambda_0$), placed parallel to the electric field vector of the EM wave. The current I induced by this wave is uniform along the dipole and is proportional to the electric field E . Then the $I \times B$ force on the antenna is found to be

$$f = ALcb^2\hat{a}_x + ALc\mu_0U\hat{a}_x \quad (14)$$

which is directed parallel to the direction of wave propagation. Here A is a constant related to the antenna.

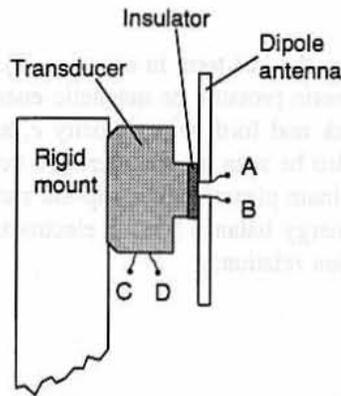


Figure 1. The conceptual force-measuring antenna. The transducer produces a voltage at the terminals CD, proportional to the force on the antenna and having the same sign. The companion wave appears at these terminals. The conventional EM wave appears at the terminals AB of the antenna.

It is now necessary to analyse the origins of such a force on an antenna placed in a magnetic field, i.e. to ask what the agent exerting this force is. If the antenna is in transmission, the agent is the generator. The force experienced by an antenna *on reception* can only arise from a momentum existing in free space. The first term on the right-hand side of equation (14) is clearly a unidirectional force associated with the momentum of the EM wave, and the observation of this force would verify the existence of this wave. The second term is associated with the companion wave. Both these forces will appear as voltages at the terminals CD of the FMA. The second term is distinguishable from the first by the bidirectional nature of the latter. We thus find that the momentum of the companion wave is as real and observable as that of the EM wave, and is not a mathematical artifact.

The above discussion leads us to a related observational consequence. On the face of it, equation (14) is valid whether the magnetic field B_0 is a distended field over many wavelengths as we have pictured, or just a local field at the antenna covering a region

much smaller than a wavelength. According to the present discussion, we do not expect the companion wave to fully develop in the latter case, and hence the force detected by an electrically small antenna (i.e. an antenna having a physical size much smaller than the wavelength) should be different in the two cases. From conventional electromagnetic theory, however, it is not immediately clear why this should be the case. We are thus left with a puzzling but testable prediction with regard to the companion wave. This test could be an experimental one, or it could be a theoretical proof that the $I \times B$ force on an electrically small antenna on reception is dependent on the extent of the magnetic field. The crux of such a proof may be the fact that even for an electrically small antenna, the effective collecting aperture (i.e. the area over which the antenna intercepts the incoming radiation) still has a dimension comparable to a wavelength [21].

5. Companion wave communication

We consider two points along the x axis: a source point S and an observation point O . An FMA is located at the point O . If the EM wave is modulated with the signal at S , then this signal will appear in both terminals AB and terminals CD of the FMA. Now we wish to study the possibility of similar information transfer through the companion wave.

At the source point S , let us modulate the static magnetic field B_0 with a superimposed 'signal' $\Delta B = \Delta B_0(x) \sin vt$, where the function $\Delta B_0(x)$ is non-zero only in the source region and is zero at the observation point O . There will be an electric field associated with the time-varying magnetic field, but we assume that the variation is slow enough that no electromagnetic radiation is emitted. For the present theoretical discussion, it is not necessary to specify a source mechanism except to note that this mechanism is continually injecting energy into the source volume. Now, replacing B_0 by $B_0 + \Delta B$ in equation (8), we find an additional flow term corresponding to ΔB :

$$S_{xs} = (c/\mu_0)\Delta B_0(x)b_0 \sin vt \sin(\omega t - k_0x)\hat{a}_x. \quad (15)$$

Clearly, this can represent a unidirectional flow of energy out of the source region. This means that the signal ΔB propagates out of the source region. The mechanism is a spreading of magnetostatic oscillations, and represents a mode of energy propagation that is different from EM waves. This signal will appear only at the terminals CD of the FMA at O as a modulation of the momentum of the 'carrier' companion wave.

6. Applicability considerations

The foregoing discussion may now be made somewhat concrete. We note now that piezoelectric and acoustoelectric transducers today can measure ultrasonic vibrations at frequencies ranging to about 10 MHz while capacitive transducers range to well above 100 MHz [22]. For a numerical estimation of the magnitude of the force on an antenna due to the companion wave, suppose that our test dipole is lossless and is connected to a matched load. Its radiation resistance is $R = 80\pi^2(L/\lambda_0)^2$, so that the current is $I = V/2R \approx EL/2R$, where V is the voltage induced on the antenna [21]. The force due to the companion wave is $f_c = ILB_0$. From these relations, we find

$$f_c \approx 6 \times 10^{-4} EB_0\lambda_0^2.$$

As an example, for an antenna in the earth's magnetic field ($B_0 \sim 0.5 \times 10^{-4}$ T), and subjected to an EM wave of flux density 1 W m^{-2} ($E_0 \approx 30 \text{ V m}^{-1}$) at 1 MHz, the peak value of f_c is about 0.1 N. If the mass of the antenna is 1 kg, this is equivalent to an acceleration of $0.01g$ ($g =$ the acceleration due to gravity), which should be detectable. One may then speculate on whether or not companion wave communication in the earth's magnetic field is a feasible technique.

7. Remarks

We return finally to equation (2). Our discussion makes it clear now that if equation (1) contains the essence of EM waves, equation (2) contains the essence of VEI. Thus, even though a derivative of equation (1), equation (2) has its own special import. The clue to that import is contained in the dielectric force we discussed at the outset.

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References

- [1] Stratton J A 1941 *Electromagnetic Theory* (New York: McGraw-Hill) ch 2
- [2] Einstein A and Laub J 1908 *Ann. Physik.* **26** 541
- [3] Minkowski H 1908 *Nachr. Ges. Wiss. Göttingen* **53**
- [4] Abraham M 1909 *Rend. Circ. Matem. Palermo* **28** 1
- [5] Brevik I 1979 *Phys. Rep.* **52** 133 (review paper)
- [6] James R P 1968 *Proc. Nat. Acad. Sci.* **61** 1149
- [7] Walker G B and Lahoz D G 1975 *Nature* **253** 339
- [8] Walker G B, Lahoz D G and Walker G 1975 *Can. J. Phys.* **53** 2577
- [9] Walker G B and Walker G 1976 *Nature* **263** 401
- [10] Israel W 1977 *Phys. Lett.* **67B** 125
- [11] Walker G B and Walker G 1977 *Nature* **265** 324
- [12] Walker G B and Walker G 1977 *Can. J. Phys.* **55** 2121
- [13] Lahoz D G and Graham G M 1979 *Can. J. Phys.* **57** 667
- [14] Lorrain P 1980 *Can. J. Phys.* **58** 683
- [15] Lahoz D G and Graham G M 1989 *Phys. Rev. Lett.* **42** 1137
- [16] De B R 1979 *Phys. Fluids* **22**(1) 189
- [17] De B R 1979 *Astrophys. Sp. Sci.* **62** 255
- [18] De B R 1980 *Phys. Fluids* **23**(2) 408
- [19] De B R 1988 *Astrophys. Sp. Sci.* **144** 99
- [20] Jackson J D 1972 *Classical Electrodynamics* (New York: Wiley) ch 6
- [21] Kraus J D 1950 *Antennas* (New York: McGraw-Hill) ch 3
- [22] Fitting D W and Adler L 1981 *Ultrasonic Spectral Analysis for Nondestructive Evaluation* (New York: Plenum) pp 50-3