

Quantum electrodynamics and experiment demonstrate the non-retarded nature of electrodynamical force fields

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Abstract

In quantum electrodynamics, the quantitatively most successful theory in the history of science, interchange forces obeying the inverse square law are due to the exchange of space-like virtual photons. The fundamental quantum process underlying applications as diverse as the gyromagnetic ratio of the electron and electrical machinery is then Møller scattering $ee \rightarrow ee$. Analysis of the quantum amplitude for this process shows that the corresponding interchange force acts instantaneously. This prediction has been verified in a recent experiment.

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One of the most remarkable developments of physical science during the 20th Century was the advent of quantum electrodynamics (QED) [1] born of the fusion of quantum mechanics and special relativity theory, QED has two important aspects; the first is the remarkable quantitative success of the theory –an essentially perfect description of nature within its domain of applicability. The second is its role as a ‘model theory’ from which the standard model of particle physics was later developed by attempting to describe the weak and strong interactions by quantum field theories in a similar way as the electromagnetic interaction is described by QED. A recent illustration of the former aspect of QED is the remeasurement at Harvard University of the gyromagnetic ratio of the electron [2]:

$$\frac{g_e^{exp}}{2} = 1.00115965218085(76)$$

The measurement uncertainty in the last two figures is indicated in parentheses. The accuracy of the measurement is 7.6 in 10^{13} , a six-fold improvement on the previous best measurement performed at the University of Washington in 1987 [3] for which Hans Dehmelt was awarded a part of the 1989 Nobel Prize in Physics. The electron gyromagnetic ratio is the most precisely measured physical quantity, to date, in the history of science. The QED prediction for g_e is [4]:

$$\frac{g_e^{thy}}{2} = 1.00115965218875(766)$$

Where the measured value of the fine structure constant: $\alpha = 1/137.036\dots$ derived from spectroscopy of the Rubidium atom [5], is used in the prediction. Theory and experiment for g_e agree at about one part in 10^{11} [4]:

$$\frac{g_e^{exp}}{2} - \frac{g_e^{thy}}{2} = -7.9(7.7) \times 10^{-12}$$

Assuming the correctness of the QED prediction for g_e , the Harvard measurement determines the value of α with ten times better accuracy than given by Rubidium spectroscopy, which yields the second most accurate experimental measurement of this constant.

The calculational problems which must be surmounted in order to obtain the QED prediction for g_e at the accuracy required by the latest experimental measurement, and so obtain a quantitatively meaningful comparison of theory and experiment, are formidable. However, as pointed out by Feynman, the basic physical concepts on which the calculations are based are extremely simple [6]. Three fundamental quantum mechanical amplitudes suffice to calculate the amplitude of any QED space-time process, no matter how complicated, involving only structureless charged particles and photons. These are the amplitudes for the processes:

A photon goes from place to place

A charged particle goes from place to place

A charge particle emits or absorbs a photon

Combination of these three amplitudes enables the quantum mechanical probability amplitude for any space-time process in QED –one Feynman diagram per amplitude– to be written down. Depending on the accuracy with which it is desired to obtain the prediction for the value of some measured observable, quantum mechanical superposition must be used to add the amplitudes corresponding to one or more Feynman diagrams. The accuracy of the latest Harvard measurement of g_e requires the evaluation of the amplitudes of 891 distinct Feynman diagrams containing up to five virtual photon lines.

The lowest order diagram contributing to the ‘anomaly’ $a_e \equiv g_e/2 - 1$ is shown in Fig.1a. The physical observable corresponding to this diagram is the angular frequency with which the spin vector of the electron rotates relative to the direction of its momentum vector: $\omega_a = a_e B/m_e$. The diagram of Fig.1a gives the prediction $a_e = \alpha/\pi$ [7]. The left diagram in Fig.1a shows the conventional way of drawing the diagram, where the vertical virtual photon line is identified with the (classical) uniform magnetic field B produced, in the Harvard experiment, by the solenoidal magnet of a Penning Trap (PT). In this experiment it is the gyromagnetic ratio of a single electron, e_{PT} , initially in the lowest lying cyclotron level of the PT, which is measured. In the QED description the classical magnetic field, B , is a manifestation of the exchange of virtual photons between the conduction electrons, e_I , of the solenoidal magnet and e_{PT} . This is shown in the diagram on the right in Fig.1a. The field B is the result, in QED, of the superposition of the amplitudes corresponding to each conduction electron in the magnet. Under the influence of the transverse Lorentz force generated by B , e_{PT} undergoes periodic cyclotron motion in the PT with angular frequency $\omega_c = eB/m_e$. The corresponding diagram in the QED description is shown in Fig.1b. The Lorentz force is produced by the exchange of virtual photons between e_{PT} and all the conduction electrons in the magnet. Using the technique of ‘Quantum-Jump Spectroscopy’ [8], the Harvard experiment measures separately ω_a and ω_c and derives a_e , and hence g_e , from the relation $a_e = \omega_a/\omega_c$.

The force on e_{PT} in Fig.1b, produced by virtual photon exchange, that causes the cyclotron motion of the trapped electron has the same origin, in QED, as the force which

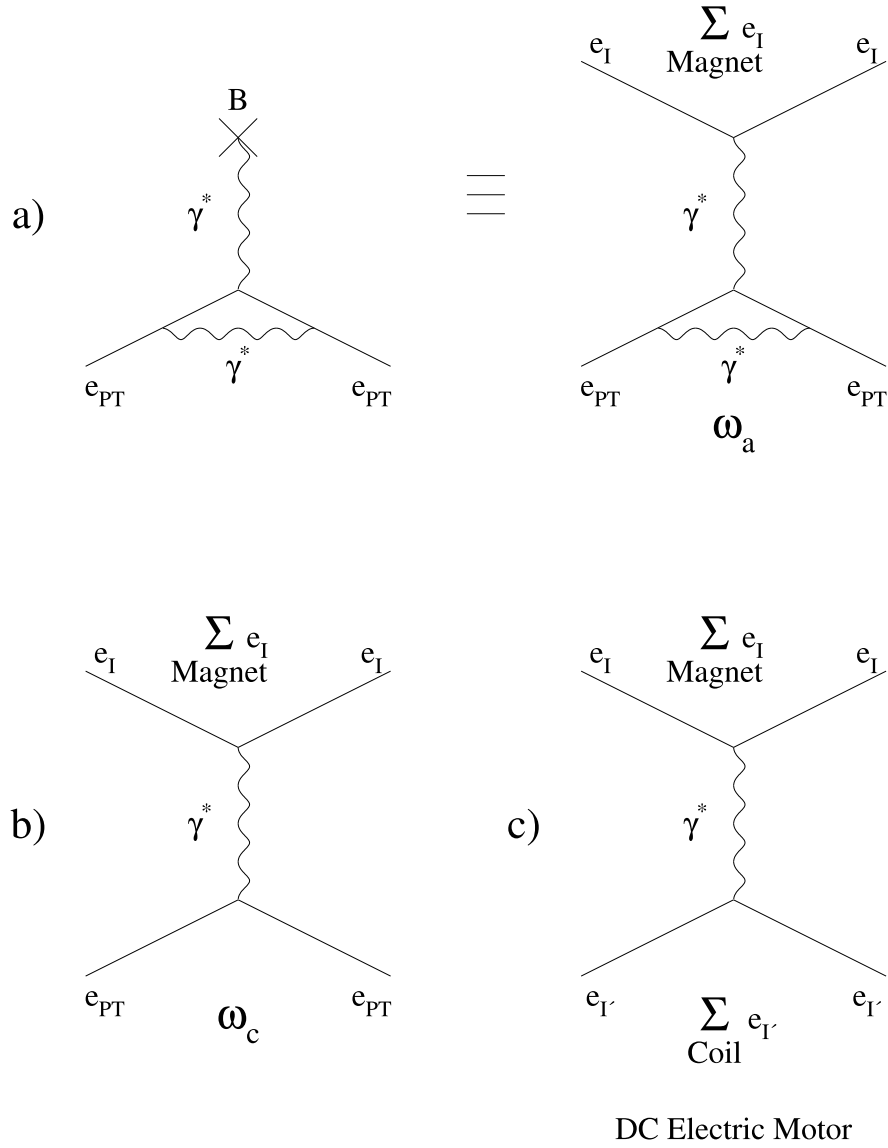


Figure 1: Lowest order Feynman diagrams contributing to: a) $a_e = g_e/2 - 1$ or the spin precession frequency ω_a , b) the cyclotron frequency ω_c in the Penning Trap of Ref. [2], c) the operation of a direct current electric motor. See text for discussion

causes the coil of a DC electric motor to rotate (Fig.1c). The electron e_{PT} is simply replaced by a conduction electron e_V in the rotating coil of the motor. Each pair of conduction electrons, one in the magnet, one in the coil, gives a contribution to the turning force of the motor.

The fundamental QED process underlying cyclotron motion of e_{PT} in the PT of the Harvard experiment, an electric motor, and indeed all inter-electron forces obeying an inverse square law in electrodynamics, including those operating in the experiment to be described below in the present paper, is then Møller scattering: $ee \rightarrow ee$ as in Fig.1b and 1c. The lowest order Feynman diagram for the process shown involves the exchange of a single ‘space-like’ virtual photon. The meaning of this appellation is explained below.

In momentum space, the invariant amplitude for Møller scattering $e_A e_B \rightarrow e_A e_B$ is [9]

$$T_{fi} = -i \int \frac{\mathcal{J}^A(x_A) \cdot \mathcal{J}^B(x_A)}{q^2} d^4 x_A \quad (1)$$

The 4-vector current \mathcal{J}^A is defined in terms of plane-wave solutions, u_i, u_f of the Dirac equation for the incoming (i) and outgoing (f) electron e_A as

$$\mathcal{J}_\mu^A \equiv -e \bar{u}_f^A \gamma_\mu u_i^A \exp[i(p_f^A - p_i^A) \cdot x_A] \quad (2)$$

The incoming electron of 4-vector momentum p_i^A emits the virtual photon at the space-time point x_A and scatters with 4-vector momentum p_f^A . The factor $1/q^2$ in (1) is the amplitude, in momentum space, for the virtual photon to propagate between points on the trajectories of e_A and e_B , q being the 4-vector momentum of the virtual photon. The integral over $d^4 x_A$ in (1) in conjunction with the exponential functions in the currents (equivalent to Dirac- δ functions) ensures energy-momentum conservation in the scattering process. In the overall center of mass (CM) frame, energy-momentum conservation requires that the energy, but, in general, not the momentum, of the virtual photon vanishes. This implies that in the CM frame:

$$q^2 = q_0^2 - |\vec{q}|^2 \rightarrow -|\vec{q}|^2 \quad (3)$$

The virtual photon is then ‘space-like’ because its 4-momentum squared is negative. If an object has positive 4-vector momentum squared, its 4-vector momentum is ‘time-like’ and a positive mass may be assigned to it. All ponderable physical objects in the real world have such time-like 4-vector momenta. As pointed out by Einstein in 1905 [10], the speed of an object with a time-like 4-vector momentum is less than that of light. However, as just shown, the virtual photons exchanged in Møller scattering are not time-like. Using (3), (1) may be written, in the CM frame, as:

$$T_{fi} = i \int \frac{\mathcal{J}^A(x_A) \cdot \mathcal{J}^B(x_A)}{|\vec{q}|^2} d^4 x_A \quad (4)$$

The Fourier transform:

$$\frac{1}{|\vec{q}|^2} = \frac{1}{4\pi} \int \frac{d^3 x e^{i\vec{q} \cdot \vec{x}}}{|\vec{x}|} \quad (5)$$

enables the invariant amplitude to be written as a space-time integral [11]:

$$T_{fi} = \frac{i}{4\pi} \int dt_A \int d^3 x_A \int d^3 x_B \frac{\mathcal{J}^A(\vec{x}_A, t_A) \cdot \mathcal{J}^B(\vec{x}_B, t_A)}{|\vec{x}_B - \vec{x}_A|} \quad (6)$$

It can be seen that the momentum-space photon propagator $1/|\vec{q}|^2$, the value of which is fixed by the electron scattering angle, corresponds, in space-time, to the exchange of an infinite number of virtual photons travelling between all spatial positions \vec{x}_A and \vec{x}_B on the trajectories of the electrons e_A and e_B . Eqn(6) shows that *each virtual photon is both emitted and absorbed at the same instant, so that the corresponding force is transmitted instantaneously*. The same conclusion follows from considerations of relativistic kinematics. The magnitude of the velocity of an object with momentum p and energy E is $v = pc^2/E$. In the CM frame of Møller scattering, the virtual photon has $p > 0$ for any non-zero scattering angle, but always $E = 0$. Therefore v is infinite and the corresponding interchange interaction instantaneous.

It has been previously noticed that the ‘Coulomb interaction’ associated with the temporal components of the currents \mathcal{J}^A and \mathcal{J}^B (exchange of ‘longitudinal’ virtual photons) is instantaneous, but it is usually stated that the interaction mediated by the spatial components of the currents (exchange of ‘transverse’ virtual photons) is transmitted at the speed of light [12]. The arguments presented above show instead that the whole interchange interaction is instantaneous in the CM frame.

QED provides a simple explanation for Coulomb’s inverse square law of force, which, in this theory, is mediated by the exchange of virtual photons. Geometrical considerations and conservation of particle number implies that, if a force is proportional to the number of some exchanged particles, it must decrease as the inverse square of the distance from the source. An example is the force of radiation pressure due to real photons emitted from the Sun. Quantum mechanics, in general, modifies this prediction. For a virtual particle of pole mass m propagating over a large space-like interval $\Delta x > \Delta t$ the amplitude for displacements Δx , Δt (the space-time propagator) takes the form: $\exp[-m\sqrt{(\Delta x)^2 - (\Delta t)^2}]$ [13]. Thus the range of the force is exponentially damped if m is non zero. Since, however, a photon has vanishing pole mass, no damping occurs for the case of space-like virtual photons, so the inverse square law is expected to hold. The same argument applies to the propagator of any other massless, particle, independantly of its interactions; e.g. exchange of a space-like virtual massless graviton could explain the inverse square law of the gravitational force.

The above predictions of QED: an instantaneous interchange interaction and the Coulomb inverse square law in electrostatics, have been used, in conjunction with relativistic invariance and Hamilton’s Principle, to formulate a classical theory of interchange forces without *a priori* introduction of any classical field concept [11]. The dynamical equations of this theory, Relativistic Classical Electrodynamics (RCED) are derived by Hamilton’s Principle from the Lorentz-invariant Lagrangian:

$$L(x_1, u_1; x_2, u_2) = -\frac{m_1 u_1^2}{2} - \frac{m_2 u_2^2}{2} - \frac{j_1 \cdot j_2}{c^2 \sqrt{-(x_1 - x_2)^2}} \quad (7)$$

that describes the electromagnetic interaction between two charged objects O1,O2 with space-time, velocity and current 4-vectors x , u and j respectively, and Newtonian mass m . The fields and potentials of Classical Electromagnetism (CEM), are all derived, as well as relativistic corrections to them, by mathematical substitution, in the dynamical equations obtained by inserting the Lagrangian (7) into the Lagrange equations that follow from Hamilton’s Principle. Also predicted are the Faraday-Lenz law, the Biot and Savart law and the Lorentz force law. The complete relativistic description of the

effect of inverse square interchange forces between O1 and O2 is provided by the ‘fieldless’ first-order differential equations:

$$\frac{d\vec{p}_1}{dt} = \frac{q_1}{c} \left[\frac{j_2^0 \vec{r} + \vec{\beta}_1 \times (\vec{j}_2 \times \vec{r})}{r^3} - \frac{1}{cr} \frac{dj_2}{dt} - \vec{j}_2 \frac{(\vec{r} \cdot \vec{\beta}_2)}{r^3} \right] \quad (8)$$

$$\frac{d\vec{p}_2}{dt} = -\frac{q_2}{c} \left[\frac{j_1^0 \vec{r} + \vec{\beta}_2 \times (\vec{j}_1 \times \vec{r})}{r^3} + \frac{1}{cr} \frac{dj_1}{dt} - \vec{j}_1 \frac{(\vec{r} \cdot \vec{\beta}_1)}{r^3} \right] \quad (9)$$

where $r \equiv |\vec{x}_1 - \vec{x}_2|$, $\beta \equiv v/c$. On neglecting relativistic corrections of $O(\beta^2)$ and higher, these equations are equivalent to Coulomb’s law, the Biot and Savart Law and the Lorentz force law of CEM.

Not described by (8) and (9) are the effects of *real* photons propagated at the speed of light, c . The corresponding *radiative* electric and magnetic fields, to be contrasted with the *force fields* implicit in (8) and (9), are produced by accelerated charges and have an r^{-1} dependence, instead of the r^{-2} dependence of the force fields. On solving the coupled differential equations (8) and (9), for the particular case of circular Keplerian orbits of two equal and opposite charges [11], it is found that the r^{-1} dependent terms in (8) and (9), although also containing acceleration factors, do not describe radiation of real photons, but rather the modification of the masses of the objects due to their mutual electromagnetic interaction.

In any experiment where source charges are accelerated and the effect on test charges is observed, contributions of two distinct types are therefore to be expected:

- (i) The effect of instantaneous *force fields*, mediated in QED by the exchange of space-like virtual photons, with r^{-2} dependence.
- (ii) The effect of *radiation fields*, mediated by propagation at the speed of light of real (on shell) photons, with with r^{-1} dependence.

Because of the rapid fall-off with distance of the force fields, special experimental arrangements are necessary for their detection. Remarkably, it is only this year, more than a century after Hertz’ experiment [14] in which ‘electromagnetic waves’ were discovered, that the results of a new one sensitive to the temporal properties (instantaneous or retarded) of the force fields has been published [15]. This experiment is now briefly described.

In essence, the experiment is a repetition of the Hertz experiment using modern electronics to detect and visualise the signals, and probing small separations of the emitting and receiving antennas in order to be sensitive to the force fields with r^{-2} dependence (called by the authors of Ref. [15], ‘bound fields’). The emitting (EA) and receiving (RA) antennae are essentially one-turn circular coils of radius 5cm and depths 5cm(EA) and 10cm(RA) consisting of 1mm thick copper sheet, and placed in the same horizontal plane, with centers separated by the distance R . The EA is activated by discharging a capacitor C , in series with the EA of inductance L , with the aid of a spark gap, to generate a pulsed harmonically varying current with angular frequency $\omega = 1/\sqrt{LC} = 7.4 \times 10^8$ rad/sec. The associated time-varying magnetic field induces a current in the RA which is displayed as a temporal signal on a 500Mhz digital oscilloscope.

Neglecting relativistic corrections, the magnetic field produced by a small single-turn coil of surface area ΔS containing a current I at a large distance R from the coil in the

plane of the latter is (see the Appendix of Ref. [15]):

$$\vec{B} = \vec{B}_v^{force} + \vec{B}_c^{rad} = \frac{\Delta S}{4\pi\epsilon_0 c^2} \left\{ \frac{[\dot{I}]_v}{R^3} + \frac{c}{v} \frac{[\dot{I}]_v}{cR^2} + \frac{[\ddot{I}]_c}{c^2 R} \right\} \hat{k} \quad (10)$$

where the dot denotes a time derivative and the unit vector \hat{k} is perpendicular to the plane of the coil. The terms with R^{-3} and R^{-2} dependence describe the effects of the force field, and are derived from the Biot and Savart law, that with R^{-1} dependence is a radiation field. The square brackets indicate retardation of the enclosed quantity; that is, it is evaluated at the time $t - R/v$ for the force field and $t - R/c$ for the radiation field. In CEM it is usually assumed that $v = c$, whereas QED predicts that $v = \infty$. In the experiment, v is assigned an arbitrary value in (10) and measured by comparing the prediction of this formula with the experimental data.. The Faraday-Lenz law predicts that the time varying current signal, $\epsilon_v(t)$, in a parallel test coil, also of surface area ΔS , placed in the field \vec{B} is:

$$\epsilon_v(t) = \frac{(\Delta S)^2}{4\pi\epsilon_0 c^2} \left\{ \frac{[\dot{I}]_v}{R^3} + \frac{c}{v} \frac{[\dot{I}]_v}{cR^2} + \frac{[\ddot{I}]_c}{c^2 R} \right\} \quad (11)$$

The harmonic variation of the magnetic field produced by EA implies that (11) may be written as:

$$\epsilon_v(t) = \epsilon_0 \left[-\frac{\sin \omega(t - R/v)}{R^3} + \frac{\omega \sin[\omega(t - R/v) - \pi/2]}{vR^2} + \frac{\omega^2 \sin \omega(t - R/v)}{c^2 R} \right] \quad (12)$$

It can be seen that the phases of the force field contributions in the first two terms are different to that of the radiation field contribution in the third term on the right side of (12). Since the phase difference between the force field and radiation field contributions depends on both R and v , the value of v can be determined by measuring the R dependence of the signal function $\epsilon_v(t)$. In practice this is done by measuring the R dependence of the time difference, Δt , between the first zero crossing points of the total signal $\epsilon_v(t)$ and a reference signal, $\epsilon^{ref}(t)$, provided by measuring $\epsilon_v(t)$ at large distances where only the radiative contribution remains:

$$\epsilon^{ref}(t) \equiv \epsilon^{rad}(t) = \epsilon_0 \frac{\omega^2 \sin \omega(t - R/v)}{c^2 R} \quad (13)$$

and extrapolating it back to the short distance region. In order to reproduce correctly the experimental conditions, the elementary formula (11) was numerically integrated over the surfaces and depths of the EA and the AR.

The measured results for Δt as a function of R , in comparison with predictions for different values of v , are shown in Fig.2. It can be seen that the case of conventionally retarded force fields, as in CEM, is completely excluded by the measurements, but that good agreement is found for the case $v \geq 10c$, which is essentially the same as the QED prediction $v = \infty$. Yet again a prediction of QED is in perfect agreement with experiment!

The physical paradigm of ‘causality’ –that no physical influence can propagate faster than the speed of light – universally accepted in physics in the second half of the 19th Century and the whole of the 20th– is thus in contradiction both with the prediction of QED and the experimental results shown in Fig.2 –the force fields generated by the EA

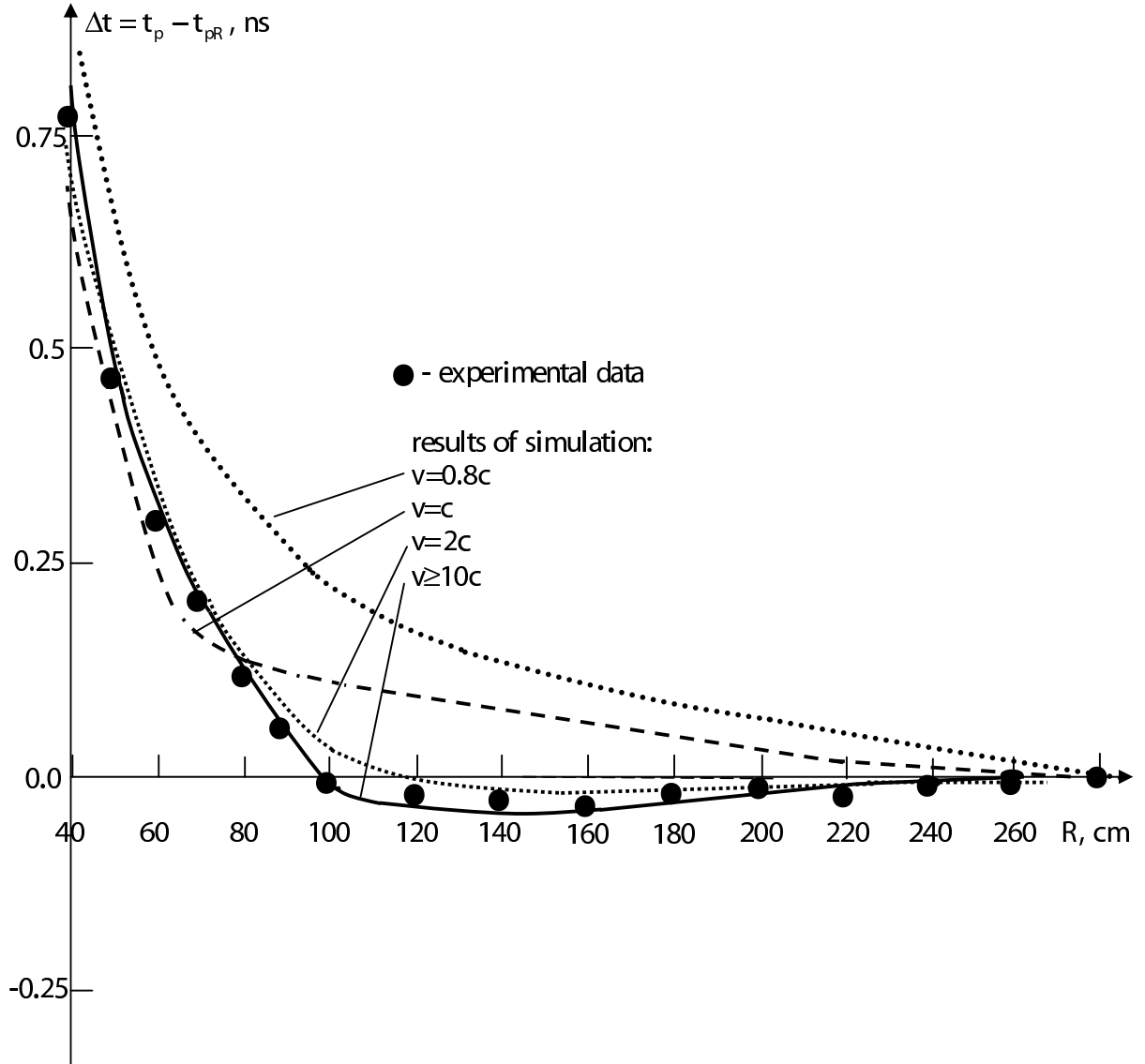


Figure 2: Measured values of the difference, Δt , between the first zero crossing times of the signals $\epsilon_v(t)$ and $\epsilon^{ref}(t)$, in comparison with simulations for different values of v from Ref.[15]. The measurement error on each experimental datum is $\simeq 0.02\text{ns}$. The prediction $v = c$ of conventional CEM is completely excluded whereas the data are consistent with the QED prediction $v = \infty$ which is indistinguishable from the $v \geq 10c$ curve shown.

arrive at the RA before the radiation fields, in QED the virtual photons arrive before the real ones. Einstein's argument in the 1905 special relativity paper [10] that 'causality' follows from special relativity is correct, only insofar as 'information' is transmitted by objects with time-like 4-momentum vectors, However the virtual photons that manifest as the force fields of CEM or RCED have space-like, not time-like 4-vectors. Any particle with a time-like 4-momentum vector is tachyonic in nature –the speed of light is a lower, not an upper, limit on its speed.

It is interesting to notice that Hertz' paper [14] reporting the discovery of electromagnetic waves propagating with a velocity 'akin to that of light' also showed data, taken close to the source, that was consistent with an infinite propagation speed for the associated signal [16, 17]. That no comment was made on this data in the conclusions of Ref. [14] is the more surprising, given that two types of electric force, one instantaneous, the other propagating at the speed of light were proposed in the electromagnetic theory of Hertz' mentor, Helmholtz [16].

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