Comments on "What the Electromagnetic Vector Potential Describes" by E. J. Konopinski

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Abstract

The seminal paper on the meaning of the vector potential by E. J. Konopinski is revisited. The full significance of this work has not been generally recognized to date. We first briefly review Konopinski's findings and show that many of his key results can be obtained from a simpler and more familiar approach. We then discuss the additional implications of his analyses, which were overlooked by Konopinski himself.

A review of current textbooks and articles on classical electromagnetism indicates that full significance of Konopinski's groundbreaking analyses[1] of the vector potential has been ignored to date. Konopinski's main result is that the vector potential represents a real field whose properties are directly measureable; hence, it should no longer be viewed as a "mathematical convenience" . Unfortunately the vector potential as a "mathematical convenience" remains the entrenched view, and is the usual basis for stating that the divergence of the vector potential is undefined. The aim of this note is to alert the scientific community of the full implications of this important paper by a distinguished physicist.

Konopinski demonstrates the fallacy of the generally held view that the vector potential A has no physical meaning in classical electromagnetism. His paper [1] follows up on Feynman's complaint [2] that a bias exists regarding the vector potential. Konopinski attributes this bias to erroreous notions arising from the gauge concept. By contrast, it is generally recognized from the Bohm-Arhonov effect [2] that A has physical meaning as a real "field" in quantum mechanics

The Coulomb potential φ has never been subjected to any such bias about its reality as a field. Its reality as a stored potential energy of a unit test charge in an existing field is generally accepted. Invoking the conservation of energy, one can, in principle, experimentally map out a scalar field, at least for a static or quasi-static case. So the scalar potential has physical meaning, and its magnitude at all points in space can be determined experimentally, which satisfies the requirement for treating the scalar potential as a real field.

Konopinski's approach is to show that the generalized momentum, $p = Mv + qA$, is a conserved quantity (in static or quasi-static cases) along any path where the gradients of A and φ vanish. Any change in A will produce a compensating change in the charged particle momentum Mv to preserve the conservation of generalized momentum along that path. Thus, A can be viewed as the momentum "stored" in the system comprised of a of a unit test charge in an external magnetic field.

Konopinski then uses the same infinite solenoid example as in the Bohm-Ahronov effect to illustrate how one can, in principle, experimentally measure A at all points in space. By applying a series of concentric rings around the solenoid, each with a unit charge on a sliding bead, one can measure A for all points in space and time by monitoring the associated changes in the bead momentum Mv arising from solenoid current changes and applying conservation of momentum (conservation of angular momentum in this case). Thus, the vector potential satisfies the criteria for a real field: A has physical meaning and its characteristics are experimentally accessible at every point in space. One of Konopinski's major accomplishments is that both classical and quantum mechanics are now on the same footing regarding the primacy of the potential formulations. This difference in views of the reality of A in classical and quantum physics was an inconsistency that had to be eliminated.

To complement Konopinski's more general development, we next offer a simple, alternative approach using a more elementary formulation. The magnetic field B is given by,

$$B = \nabla \times A \ , \tag{1}$$

and the total electric field, E , (sum of Coulomb and induced fields) is given by,

$$E = E_C + E_I = -\nabla\varphi - \partial A/\partial t \tag{2}$$

Although never stated explicitly in the literature, the relationship for E_I in Eq. 2,

$$E_I = -\partial A/\partial t \ , \tag{3}$$

is actually an expression of Faraday's induction law, which, we suggest, should generally be framed as such to properly recognize its stature. This would also serve to better reflect Konopinski's conclusion regarding the physical meaning and measureability of A . Faraday's law is usually expressed in terms of a closed line integral of the induced electric E_I which limits its utility to cases of high symmetry. By contrast, Eq. 3 is completely general. Furthermore, the electric field formulation relies on the outmoded "action at a distance" concept, made obvious here by the fact that $B = 0$ everywhere outside the solenoid. Explicitly recognizing Eq.3 as a fundamental law clearly illustrates that the vector potential A is real and meaningful because its time derivative is real and meaningful. In other words, if E_I is a real field, then A must be a real field because it is the source of E_I at every point in space (including regions where B=0).

Using this form of the law of induction (Eq. 3), one can experimentally determine A with the same solenoid setup (without explicitly invoking generalized momentum conservation) by increasing the solenoidal current from zero to its value at time t while concurrently monitoring the change in the bead momentum. There are no external Coulomb fields, and no gradients in A along the ring, so the force on the unit charged bead of mass M is given by $F = d(Mv)/dt = -\,\partial A/\partial t = -dA/dt$, giving $A(t) = -Mv(t)$. In this alternative approach, A is given physical meaning through Faraday's law of induction, and its measureability is demonstrated by using Konopinski's solenoid apparatus.

The next point of this Note relates to a major implication of Konopinski's analyses that he overlooked. In deriving the analytic expression for A outside the infinite solenoid, he invokes the Coulomb gauge, $\nabla \cdot A = 0$, in the usual manner as a "convenient choice". So, on the one hand, A is physically real and measureable, while on the other, $\nabla \cdot A$ is completely arbitrary. The generally accepted view that $\nabla \cdot A$ is completely arbitrary is an absurdity if Konopinski is correct, because establishing A for all points in space and time establishes its divergence. The resolution of this apparent paradox is found by recognizing a general carelessness in applying the Lorenz gauge concept . Recall that one can always transform the unprimed, real, physically meaningful potentials to the primed variables according to the Lorenz prescription,

$$A' = A + \nabla\vartheta; \;\; \varphi' = \varphi - \partial\vartheta/\partial t, \qquad (4)$$

where ϑ is an arbitrary function of time and space. Although A' and φ' are physically meaningless, the Lorenz transformation leaves E in Eq. 2 unchanged, so it retains its physically meaning. So, it is A' whose divergence is arbitrary in the Lorenz gauge, not A . There is no choice for $\nabla \cdot A$.

Another frequently repeated form of the error used to justify the Coulomb gauge in the literature is that A is defined by Eq.1, so one can always add any gradient function of space and time to A without altering the physics of the problem. This is false. The vector potential is defined by *both* Eq. 1 and Eq. 3. One must apply the proper Lorenz transformation (Eq. 4) to both A and φ to preserve the correct physics. The fundamental reason that $\nabla \cdot A = 0$ in the present case is that all solenoidal fields are solenoidal. There are no sources or sinks for E_I , so there can be no sources or sinks for $\partial A/\partial t$ in Eq. 3, which, in turn, means that $\nabla \cdot A$ must be zero. Physics dictates $\nabla \cdot A$. It is not a matter of mathematical convenience.

Any doubts about $\nabla \cdot A$ can be removed by performing Konopinski's experiment. The betatron offers a sufficiently close experimental arrangement so it can be viewed as a device for experimentally determining the vector potential. The fact that betatrons perform as designed is direct experimental proof of the validity of the analytic expressions derived for A under the assumption that $\nabla \cdot A = 0$. So, the betatron offers direct experimental proof that $\nabla \cdot A$ is not arbitrary.

In our view, one cannot overstate the importance of Konopinski's paper: it clarifies the fundamental meaning of the vector potential, eliminates major inconsistencies, and provides a strong argument for the primacy of the potentials over the electric field variables. (See reference 3 for further examples of improper applications of the gauge concept.)

References

1. E. J. Konopinski, Am.J. Phys. 46(5) 499(1978).
2. R.P. Feynman, R.B. Leighton, and M. Sands, The Feynman Lectures, (Addison-Wesley, Palo Alto, CA, 1965).
3. P.J. Cote and M.A. Johnson, arXiv:0912.2977 v2.