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Magnetostatic interactions between two magnetic wires

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Abstract – The results of the magnetic dipolar field in a simple set of two amorphous ferromagnetic wires of composition $Fe_{77.5}Si_{12.5}B_{15}$ placed side by side are presented. Owing to their peculiar domain structure, they could, in principle, be approximated by macroscopic magnetic dipoles, allowing the analysis of the magnetostatic field between these magnetic entities. Magnetization measurements as a function of the distance between the parallel wires were performed. Results can be explained considering the magnetostatic field created by one wire in the neighboring one. It is clearly shown that this field is responsible for changes of the reversal field of the wires, leading to the appearance of plateaux during the demagnetization process. Instead of pure dipolar model that does not fit experimental data, a multipolar model has been developed, showing a rather good agreement with the experimental results.

Introduction. – Dipolar interactions among magnetic entities have been widely studied, because they are fundamental to the progress of basic research and to the development of a number of application oriented novel magnetic devices. Advances in fabrication techniques (including chemical routes, electrodeposition and lithography) have allowed the fabrication of nanostructured systems with very interesting physical properties. In particular, it is nowadays possible to obtain controlled arrays of magnetic wires with diameters of few nanometers, which are of practical interest in the design and optimization of devices for ultrahigh-density data storage applications, for example [1]. In such systems, as in many other artificial magnetic structures, magnetostatic interactions may play a fundamental role in the magnetization reversal process and domain structures of the individual elements, which consequently would influence the overall magnetic response of the system.

The magnetic dipole is the basic entity of magnetism. Any calculation or simulation used to describe the magnetic behavior of a system employs this concept. However, in the case of real systems, either nano, micro or macroscopic, the effective magnetostatic interactions among the elements are still unknown, although they could have a strong influence on the macroscopic magnetic behaviour of the system. An intrinsic obstacle in the experimental study of magnetic interactions is the fact that it is extremely difficult to single out an individual magnetic element, even using the most sensitive magnetometric techniques. Also, the predictions of numerical simulations are intricate to compare with real systems, owing to the necessity of introducing several approximations in the modeled problem. However, a very interesting macroscopic analogous has been extensively studied, placing together several ferromagnetic amorphous wires and microwires [2]. The stray fields couple the magnetizations of neighboring wires, affecting the magnetic state of each single wire. Such systems are relatively easy to study experimentally, and, in the case of few wires, it was possible to obtain analytical solutions (by assuming a simple dipole approximation) [3]. The predicted solutions and experimental data can be compared with Monte Carlo simulations, which are required when the array is formed by a large number of wires [4].

Although an array composed of few ferromagnetic wires seems to be a quite simple problem to study and model, it is striking to realize how intricate this problem can turn out to be [5]. The complication in the study of dipolar interactions is that the magnetic fields resulting from the interaction depend on the magnetization state of each single entity, which, in turn, depends on the effective field of neighboring elements. In this context, numerical simulations using Monte Carlo algorithms have been usually employed to analyze the configuration of such systems in terms of well-established theoretical approaches, such as Ising and Heisenberg models, for example [6]. However, it is worth noting that the usually employed dipole approximation is not valid (see below), and a more complex theoretical treatment is necessary to understand the magnetic response of the system.

In the present work, a detailed study of the magnetostatic interaction between two parallel soft magnetic amorphous wires placed side by side is shown. We have investigated the variation of the interaction strength with distance, and the experimental results indicate that a simple dipolar model cannot explain the observed decay of the strength with distance. In order to explain such behavior, we have developed a multipolar model for the interaction that explains the experimental results with better accuracy.

Experimental features. – The studied amorphous wires were produced by the in-rotating water quenching technique having nominal composition $Fe_{77.5}Si_{12.5}B_{15}$ and diameter of 132 μm. Quasistatic magnetic hysteresis loops were performed at room temperature in a home-made hysteresis loop tracer with a fluxmeter. The magnetic field was applied along the wire axes and was generated by long solenoid (32 cm), while the pick-up coils were 3 cm long (to avoid detecting border effects). To carry out hysteresis loops from two parallel wires, two distinct approaches were used: when the distance between the wire centers was below 3 mm, the measurements were performed within the same pick-up coil; otherwise the measurements were obtained with each wire in a different pick-up coil.

The domain structure of each stable state contains a large domain whose magnetization reversal is responsible for the large Barkhausen jump, giving rise to the bistable magnetic behavior between two stable magnetic configurations. A closure structure appears at each end of the wire, whose length determines the critical length to the appearance of magnetic bistability. Once the critical length in amorphous wires is of the order of 8 cm [7], we have cut 10 cm long pieces of wires from a longer wire in order to observe a bistable magnetic behavior (two remanent states) in each individual sample [8].

Figure 1(b) shows the experimental hysteresis loops measured on two wires placed side by side forming a simple array with distance between the axes of the wires of around 0.5 mm. In the case of a single wire (see fig. 1(a)) the hysteresis curve exhibits a typical square loop, with characteristic large Barkhausen jumps, as expected for high magnetostriction amorphous wires [9]. Also, a slight decrease of magnetization prior to the giant Barkhausen jumps is observed. This reduction in the magnetization is probably connected to the orientation of the domains in the closure structure and the radially oriented outer shell [10]. In the case of one single wire the magnetization reversal occurs for $|H^*| = 8$ A/m, where H^* is the switching field. In several pieces cut from the same master wire we have verified fluctuations in H^* up to 5 A/m. Such fluctuations are mostly due to the induction of anisotropies (mainly magnetoelastic) when the samples are cut from the original piece. This fact was observed in several pieces of wires with length varying between 8 and 12 cm. This effect can be even observed in fig. 1(a), where differences in the values of magnetization are found slightly before the respective giant Barkhausen jumps for each direction of the applied magnetic field.

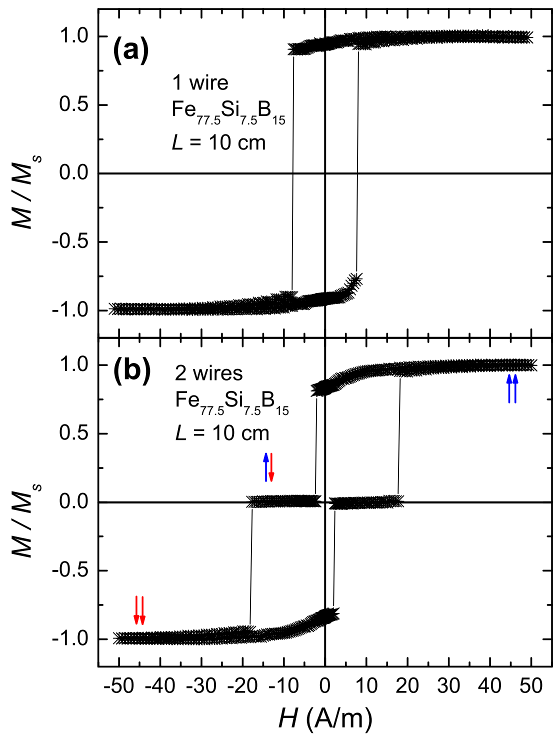

Fig. 1: Hysteresis loop for the $Fe_{77.5}Si_{7.5}B_{15}$ amorphous wire: (a) single wire, (b) two parallel wires (arrows indicating the magnetic configuration).

We have also performed a study of fluctuations of H^* in a single piece of wire. The histogram of the values obtained from 41 hysteresis loops is shown in fig. 2, showing that the amplitude of this fluctuation is around 0.4 A/m, indicating that the domain configuration is slightly changed after each cycle, giving rise to small fluctuations in the switching field. Also, it is worth mentioning that the position of the wires inside the pick-up coils is critical in these measurements, and small

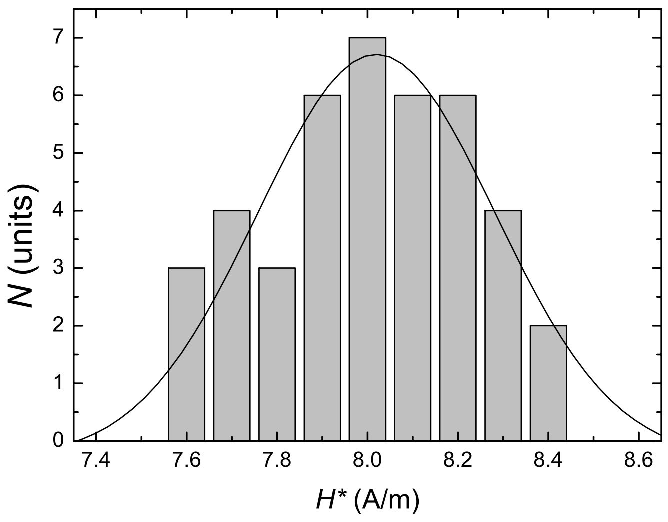

Fig. 2: Histogram of the switching field values obtained from 41 hysteresis loops.

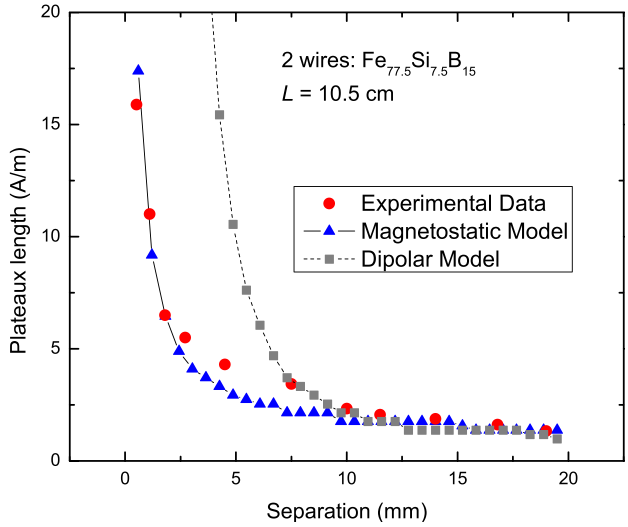

Fig. 3: Measured plateau length as a function of the interwire separation. Pure dipolar model (squares) and magnetostatic model (triangles) including all multipolar terms.

variations could cause fluctuations in the switching fields. Such fluctuations have a relevant role in the study of dipolar interactions among several wires.

In the case of a couple of wires separated by 0.5 mm, the hysteresis loops exhibit two clear Barkhausen jumps and plateaux at zero magnetization (see fig. 1(b)). Such plateaux correspond to the configuration of two wires with opposite magnetization directions. It is worth noticing that the first jump occurs at magnetic fields lower than the value of H^* of a single wire, while the second one occurs for fields larger than H^*. These reversal fields were named H_2^{i} and H_2^{ii}, and their values are 2.3 and 18.2 A/m, respectively, for positive demagnetizing field. The splitting of H^* into two reversal fields has its origin in the dipole-dipole interaction between the wires [11]. Varying the number of wires, the hysteresis loops correspondingly exhibit several steps on the demagnetization, each one corresponding to the reversal of the magnetization of a single wire [2,3].

Theoretical model. – The standard theoretical model for the description of the interaction of two wires has been the dipolar interaction with some phenomenological modifications [12]. In such model, one considers that beyond the applied magnetic field, $\mathbf{H}$, each wire feels the influence of a dipolar field, $\mathbf{H}_{i,j}$, whose origin is due to the presence of the other wire, where $\mathbf{H}_{i,j}$ is the field of the wire i over the wire j. So, if one considers each wire as a single dipole, the resulting dipolar field $\mathbf{H}_{i,j}$ would be given by: $\mathbf{H}_{i,j} = K_n \mathbf{M}_i$ being K_n a geometric factor which depends on the distance between interacting wires, the subscript n simply denotes the relative distance between the wires and $\mathbf{M}_i$ the magnetization of the i-th wire [3]. Therefore, for two wires one can easily obtain the mutual dependence produced by the dipole-dipole interaction through the expression $\mathbf{M}_k(\mathbf{H}+\mathbf{H}_{j,k})$, with $k=(1,2)$ and $j \neq k$. However, such a simple dipole approximation is not valid when dealing with interacting wires which are closely placed forming a linear array. This is clearly observed when the dipolar model is used to fit the effects of magnetostatic fields as a function of the distance. In such case, strong discrepancies appear, as can be clearly seen in fig. 3, which shows the plateaux width Δ as a function of the separation between two wires.

Considering the above mentioned discrepancies, we have developed a new model to better consider the magnetostatic interactions among magnetic wires. Let us consider the scalar magnetic potential of a cylinder characterized by a radius R and a height L, assuming that the magnetization is uniform and described by $\mathbf{M} = M_z \hat{\mathbf{z}}$, where z is the cylinder axis [13]. The potential outside the body is given by

$$U(\mathbf{r}) = \frac{\gamma_B}{4\pi} \int_{S(V)} \frac{\mathbf{M}(\mathbf{r}') \cdot \mathrm{d}\mathbf{S}'}{|\mathbf{r}-\mathbf{r}'|} - \frac{\gamma_B}{4\pi} \int_{V} \frac{\nabla' \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \mathrm{d}^3 r'. \quad (1)$$

where γ_B is a constant for the different system of units (in the international system of units $\gamma_B = 1$, and for cgs system $\gamma_B = 4\pi$) [14]. Note that the second term on the righ-hand side of eq. (1) vanishes, because the magnetization field is constant; furthermore, in the surface integral of eq. (1) the only contributions arise from the upper and lower bases of the cylinder: the upper circle is located at $z = L$ and the lower one at $z = 0$. Due to the symmetry of the problem, we used the adequate type of the cylindrical kernel for the integral [15], and after few manipulations, one finds that the integral expression for the scalar potential is given by

$$U(\mathbf{r}) = \frac{\gamma_B M_z R}{2} \int_0^\infty \frac{\mathrm{d}k}{k} J_1[kR] \times \left(\exp[-k|z-L|] - \exp[-k|z|]\right) J_0[k\rho], \quad (2)$$

where $J_m[q]$ is the Bessel function of integer order and ρ is the axial radius of the cylindrical coordinates. It is worth noticing that in the experimental situation, R/L is well defined and it is of the order of 10^{-4}, whereas for the interwire distance D, D/L can be any value (usually $D/L < 1$). Owing to the experimental conditions, we work in the regime characterized by $R/L \ll 1$, our wires have a diameter of the order of 130 μm. In this case the potential from eq. (2) can be approximated by

$$U(\mathbf{r}) \cong \left(\gamma_B M_z R^2/4\right) \times \left(\left(\rho^2+(z-L)^2\right)^{-1/2} - \left(\rho^2+z^2\right)^{-1/2}\right). \quad (3)$$

Once the potential is known, the magnetic field is easily obtained from $\mathbf{H}(\mathbf{r}) = -\nabla U(\mathbf{r})$. Hence, it is possible to proceed to calculate the magnetostatic interaction energy, E_{12}, between two identical parallel cylinders

$$E_{12} = -\int_{V_2} \mathbf{M}_2 \cdot \mathbf{H}_1(\mathbf{r}_2)\, \mathrm{d}V_2, \quad (4)$$

where the sub-indexes label the cylinders 1 and 2, respectively. When the magnetization field is constant, the contribution of the magnetostatic energy is only due to a surface term; and it is also possible to use the integral expression of the scalar, eq. (2). Therefore, the integral expression of interaction energy between two cylinders characterized by magnetizations M_{1z} and M_{2z}, respectively, is given by

$$E_{12} = 2\gamma_B M_{1z} M_{2z} V \int_0^\infty \frac{\mathrm{d}x}{x^2}\left(1-\exp[-x]\right) \times J_0\left[(D/L)\,x\right]\left(J_1\left[(R/L)\,x\right]\right)^2. \quad (5)$$

In the case of wires one has $R/L = \alpha \ll 1$, in such case one can use that $J_1[\alpha x] \approx \alpha x/2$ and with this approximation eq. (5) can be written in a very simple form

$$E_{12} \cong \frac{\gamma_B}{2\pi}\frac{(V\cdot M_{1z})(V\cdot M_{2z})}{L^2 D}\left(1-\left(1+(L/D)^2\right)^{-1/2}\right). \quad (6)$$

We remark that this expression includes all multipolar terms of the interaction and for large separation D, each cylinder can be considered as a single dipole. Therefore, their interaction is dipolar at first order, which can be easily seen from eq. (6) in the limit $L/D \ll 1$; in such case the energy becomes

$$E_{12} \xrightarrow{L/D\ll 1} \gamma_B (V\cdot M_{1z})(V\cdot M_{2z})/\left(4\pi D^3\right)$$

This relationship describes the energy of two dipoles separated by a distance D to each other. On the other hand, when $L/D \gg 1$ one has the limit

$$E_{12} \xrightarrow{L/D\gg 1} \gamma_B (V\cdot M_{1z})(V\cdot M_{2z})/\left(2\pi D L^2\right).$$

A careful analysis of eq. (6) indicates that the dipolar approximation appears valid when D approaches the value of L, corroborating that both energy expressions should coincide when $L/D \ll 1$. Thus, when considering magnetostatic interaction among nanowires, microwires or even wires, the dipole-dipole approximation usually overestimates the actual interaction at short distances. It is worth mentioning that recently, another interesting approach to calculate the magnetostatic interaction between magnetic particles has been developed by Beleggia *et al.* [16], with similar results.

Comparison between theory and experimental data. – In order to reproduce the magnetic behavior of two wires, we have used a rectangular box model for each wire, $M = F(H_{eff}, H^*)$, where H_{eff} is the effective field on the wire and H^* is its switching field; in addition F represents the bi-valuated rectangular box function, which resembles the hysteresis cycle of one bistable wire, as shown in fig. 1(a). We have used different values for the switching fields, 7.5 A/m and 8.5 A/m, respectively (see fig. 2). The effective field on each wire is then the sum of the external field plus the magnetic field produced by the neighboring wire. Furthermore, we assumed that magnetic field produced by wire i, with magnetic moment pointing in the $+z$ direction, on wire j is in the $-z$ direction, and with a magnitude calculated from the eq. (5) divided by the magnetic moment of the wire j. Therefore doing a simple simulation starting from saturation at higher external fields we obtained the hysteresis curve of the two wires system $M_1 + M_2 = F(H_{eff1}, H_1^*) + F(H_{eff2}, H_2^*)$. Figure 4 shows the theoretical hysteresis loops. It is worth noting that the model is in fairly good agreement with the experimental data.

Let us compare the theory with the experiment depicted in fig. 3, *i.e.*, the dependence of the magnetostatic interaction with the distance between the wires. The width of the plateaux is a direct measurement of the strength of magnetostatic interaction between both wires. The spatial variation of this quantity furnishes relevant information on the nature of the dipolar fields generated by each wire [12]. Indeed, the experimentally measured decay of the interaction field is much faster than the corresponding decay predicted for the present experimental conditions in the pure dipolar model. The squares depict the result of modeling the plateaux by assuming that the two wires interact via a simple dipole-dipole term. For this curve we take as the effective magnetic volume V^* only 1% of the full volume V of each wire, this is fairly a very low value and seems to be very unrealistic. It is observed that the fit is good only at large separations. By further decreasing the effective volume V^*, the fit improves in the low separation region, but it deteriorates at larger separations. So by using only a dipole-dipole term there is no way to fit the entire separation region by using a single effective volume. The triangles depict the results obtained using the approximated full magnetostatic model. The fit

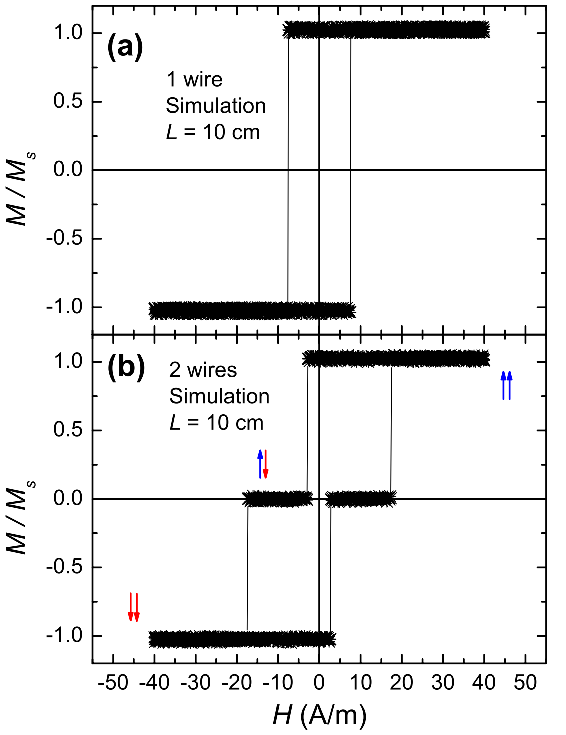

Fig. 4: Simulated hysteresis loop for one and two wires based on the box model.

is based on eq. (6), which takes into account all multipolar terms in an approximated way. Therefore, a simple model of the magnetostatic interaction between two magnetic monodomains agrees rather well with experimental data. Also, from the model it is possible to extract some physical parameters. Using the experimental value for the saturation magnetization of 1.5 T and a volume of a perfect cylinder $V = \pi R^2 L$, with $L = 10.5\,\text{cm}$ we obtain an effective core radius of $R^* = 47\,\mu\text{m}$, which is around 28% smaller than the real radius of $R = 66\,\mu\text{m}$. In fact, this can be roughly considered as the error of assuming a homogeneous magnetization in our model. Thus, the effective monodomain volume V^* is roughly 50% of the geometrical volume V of the wire, in agreement with previous theoretical calculations and complementary experimental data [7].

Although the microscopic mechanism of the domain nucleation of this kind of wires has not been taken into account, the results presented here are encouraging. Modeling a wire as a single domain with anisotropy assuming a rectangular box model for their hysteresis, is enough to understand the behavior of the magnetization with distance when placing two or more wires parallel at a given distance. The main reason, due to geometry, is that the magnetostatic interaction decays inversely with distance and not with distance to the third power as in the standard dipole-dipole approximation. The results for arrays of microwires and more realistic structure of magnetization will be presented elsewhere.

Final remarks. – The strength of magnetostatic interaction between two parallel $Fe_{77.5}Si_{12.5}B_{15}$ wires was measured for different distances between the wires. Considering each wire as a magnetic dipole, the magnetostatic dipolar field cannot describe quantitatively the interactions as a function of the distance between the wires. A more complex model concerning multipolar field contributions is presented, providing a good description of the magnetostatic interactions.

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