REPLY TO “CRITICISM OF ‘NECESSITY OF SIMULTANEOUS CO-EXISTENCE OF INSTANTANEOUS AND RETARDED INTERACTIONS IN CLASSICAL ELECTRODYNAMICS’ ” by J.D.Jackson

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In this note we show that Jackson’s criticism of our work “Necessity of simultaneous co-existence of...” is based on an inexact understanding of the basic assumptions and conclusions of our work.

In his note\(^1\) J.D.Jackson affirms that in our work\(^2\) we “make the claim that the electric and magnetic fields derived from the Liénard-Wiechert potentials for a charged particle in arbitrary motion do not satisfy the Maxwell equations”. This affirmation of J.D.Jackson does not correspond to a keynote of our work.

Let us begin with our general objections to Jackson’s criticism. Actually, one of the aims of our work was to show that the direct use (from the mathematical point of view) of the following idea of Landau and Lifshitz\(^3\) (see the quotation below) leads to a contradiction:

“\(^{\text{To calculate the intensities of the electric and magnetic fields from the formulas}}\)

\[ E = -\nabla \varphi - \frac{1}{c} \frac{\partial A}{\partial t}, \quad B = [\nabla \times A]. \tag{1} \]

we must differentiate \(\varphi\) and \(A\) with respect to the coordinates \(x, y, z\) of the point, and the time \(t\) of observation. But the formulas (63.5, Ref.3)

\[ \varphi(r, t) = \left\{ \frac{q}{(R - R_c)} \right\}_{t_0}, \quad A(r, t) = \left\{ \frac{qV}{c(R - R_c)} \right\}_{t_0}. \tag{2} \]

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express the potentials as a functions of \( t_0 \) (\( t' \) in Ref.3), \textbf{and only through} the relation (63.1) in Ref.3

\[
t_0 = t - \tau = t - \frac{R(t_0)}{c}.
\]

as implicit functions of \( x, y, z, t \). Therefore to calculate the required derivatives we must first calculate the derivatives of \( t_0 \)"

In other words, if one takes into account exclusively the implicit dependence of the potentials and fields on time \( t \), one obtains correct fields, but these fields

\[
E(r, t) = q \left\{ \frac{(R - R \frac{\nabla}{c})(1 - \frac{V^2}{c^2})}{(R - R \frac{\nabla}{c})^3} \right\}_{t_0} + q \left\{ \frac{[R \times [(R - R \frac{\nabla}{c}) \cdot \frac{\nabla}{c}]}}{(R - R \frac{\nabla}{c})^3} \right\}_{t_0},
\]

\[
B(r, t) = \left\{ \frac{R \times E}{R} \right\}_{t_0},
\]

do not satisfy the Maxwell equations. \textbf{Once more}: if, following Landau and Lifshitz aforementioned idea, one does not take into account the explicit dependence of fields on \( t \), rather only the implicit one, we can see that in this case (exclusively in this case!) fields (4) and (5) do not satisfy Maxwell equations.

In Section 4 of our work\(^2\) we showed that \textit{Faraday’s law is obeyed} if one considers the functions \( E \) and \( B \) as functions with both implicit and explicit dependence on \( t \) (or on \( x_i \)).\(^a\) That is why we do not understand why J.D.Jackson did the same in Section 4 of his work\(^1\). It seems to us that a basic reason behind Jackson’s antagonism to our work is the following: \textit{our interpretation of the explicit time-dependence as a certain manifestation of instantaneous action-at-a-distance and on the other hand the implicit time-dependence (i.e. exclusively through the relation (3)) as a well-known short-range action}. From the \textit{generally accepted}\(^b\) formal mathematical point of view our work is faultless. Let us explain this point more particularly.

In his work\(^1\) J.D.Jackson considers our expression

\[
\frac{\partial R}{\partial t_0} = -c
\]

as wrong, referring to formula \( R = r - r_0(t_0) \). The point is what one means by the operator \( \frac{\partial}{\partial t} \) and by a function \( R \) in Ref. 3. It is easy to prove that in the unenumerated set of equations before Eq.(63.6)\(^3\)

\[
\frac{\partial R}{\partial t} = \frac{\partial R}{\partial t_0} \frac{\partial t_0}{\partial t} = -\frac{RV}{R} \frac{\partial t_0}{\partial t} = c \left( 1 - \frac{\partial t_0}{\partial t} \right)
\]

\(^a\)By the way, Landau and Lifshitz in Ref.3 (we show this below) do the same, conflicting with their phrase cited above.

\(^b\)in works\(^4;5\) we show that the generally accepted point of view on the total and partial differentiation has some serious problems
Landau and Lifshitz mean by $\frac{\partial R}{\partial t}$ the total derivative and not partial one! (Recall that our index \(0\) corresponds to the index \('\) in Ref. 3). In order to obtain a value of $\frac{\partial t_0}{\partial t}$ one cannot perform the usual operation of differentiation. It is possible to calculate this derivative using a certain mathematical trick only. The authors of Ref. 3 use the fact that two different expressions of the function $R$ exist:

$$R = c(t - t_0), \quad \text{where} \quad t_0 = f(x, y, z, t), \quad (8)$$

and

$$R = \left[ (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \right]^{1/2}, \quad \text{where} \quad x_{0i} = f_i(t_0). \quad (9)$$

It is well-known from the classical analysis that if a given function is expressed by two different types of functional dependencies, then exclusively total derivatives of these expressions with respect to a given variable can be equated (contrary to the partial ones). Comparing the total derivatives of $R$ from Eq.(8) and $R$ from Eq.(9) Landau and Lifshitz obtain the corrected value of $\frac{\partial t_0}{\partial t}$. So one can see that the expression $\frac{\partial R}{\partial t}$ is the total derivative $\frac{dR}{dt}$. And from

$$\frac{dR}{dt} = \frac{d}{dt}[c(t - t_0)] = \frac{\partial R}{\partial t} \frac{\partial t}{\partial t} + \frac{\partial R}{\partial t_0} \frac{\partial t_0}{\partial t} = c \left( 1 - \frac{\partial t_0}{\partial t} \right) \quad (10)$$

one can see that $\frac{\partial R}{\partial t} = c$ and $\frac{\partial R}{\partial t_0} = -c$. However, one must not forget that these expressions are just formal mathematical equalities and they do not have any physical sense but help us to find a value of $\frac{\partial t_0}{\partial t}$. Let adduce the scheme which was implicitly used in Ref. 3 to obtain $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$: 
If one takes into account that $\partial t/\partial x_i = \partial x_i/\partial t = 0$, as a result one obtains the correct expressions for $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$.

Finally, regarding two phrases of J.D.Jackson in the Abstract and at the close of Ref. 1: “Classical electromagnetic theory is complete as usually expressed” and “Electromagnetic theory is complete in any chosen gauge”, two sufficiently authoritative physicists of 20-th century help us:

R. Feynman$^6$:

“...this tremendous edifice (classical electrodynamics), which is such a beautiful success in explaining so many phenomena, ultimately falls on its face. ...Classical mechanics is a mathematically consistent theory; it just doesn’t agree with experience. It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the ideas of Maxwell’s theory which are not solved by and not directly associated with quantum mechanics...”

W. Pauli$^7$:

“We therefore see that the Maxwell-Lorentz electrodynamics is quite incompatible with the existence of charges, unless it is supplemented by extraneous theoretical concepts” (The choice of italics was Pauli’s).

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References