Characteristics and Potential Applications of Nonlinear Left-Handed Transmission Lines

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ABSTRACT: The characteristics of nonlinear left-handed (NL-LH) transmission lines (TLs) are discussed for the first time. A LH-TL with varactor-diode series capacitance is analyzed in terms of dispersion and nonlinearity characteristics, and its voltage-wave partial differential equation is derived. Numerical responses of the NL-LH-TL to harmonic and pulse waves illustrate the conjoint effects of frequency dispersion and nonlinearity. Potential applications are suggested. © 2004 Wiley Periodicals, Inc. Microwave Opt Technol Lett 40: 471–473, 2004; Published online in Wiley InterScience (www.interscience.wiley.com).
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1. INTRODUCTION
Simultaneously nonlinear and frequency-dispersive media have been studied for long time in optics, where an adequate balance between positive nonlinearity and anomalous dispersion in Kerr materials is known to produce solitary waves, or solitons, for appropriately shaped input pulses [1, 2].

In the present contribution, we investigate the response of an LH-TL [3, 4], which is intrinsically frequency dispersive, when the line is further made nonlinear by the introduction of varactor diodes [5]. In the past, conventional, or right-handed (RH), NL-TLs were described by Lonngren and shown to support KdV solitons [6]. The question of the possible existence of solitons in an LH-NL-TL will be addressed. More importantly, interesting effects, such as pulse compression, do not require pure solitary solutions but merely the co-existence of nonlinearity and dispersion [1, 7]. We propose here an initial exploration of an LH-NL-TL in that perspective.

The availability of novel microwave pulse-shaping components could be of great use in applications such as ultra-wideband (UWB) systems [8], where data rates are directly related to pulse width, and in radar systems, also requiring very narrow pulses for high resolution.

2. NONLINEAR LEFT-HANDED TRANSMISSION LINE
A nonlinear LH-TL is represented by its incremental model in Figure 1(a). Nonlinearity is provided by a varactor diode while dispersion is an intrinsic property of the LH line. The frequency dispersion of the LH-TL is given by [3, 4]:
coefficient of the pulse and a negative frequency shift in the trailing half. Self-modulation introduces a positive frequency shift in the leading half and a positive shift in the trailing half ($\gamma > 0$).

In summary, the nonlinear LH-TL of Figure 1 exhibits anomalous frequency dispersion ($\beta'' < 0$) and negative nonlinearity ($\gamma < 0$). In comparison, optical solitons solution to the nonlinear (NLS) Schrödinger equation also require $\beta'' < 0$ but $\gamma > 0$. This could suggest that the nonlinear LH-TL does not support Schrödinger solitary waves. However, other types of solitons may be possible since a vast number of different nonlinear systems admit soliton solutions [2], and other effects such as pulse compression may also be possible.

3. WAVE EQUATION

In the long wavelength approximation ($\Delta \lambda/\lambda \rightarrow 0$), the circuit of Fig. 1 can be described by

$$i(z) = \frac{\partial}{\partial t} \left\{ \frac{C_s}{\Delta \lambda} \left[ u(z) - u(z + \Delta \lambda) \right] \right\} = -\frac{\partial}{\partial t} \left[ \frac{\partial q'}{\partial z} \right] \rightarrow \frac{\partial^2 q}{\partial z^2} + i = 0,$$

$$-u(z + \Delta \lambda) = L' \frac{\partial}{\partial t} \left[ i(z + \Delta \lambda) - i(z) \right] \rightarrow -u(z)$$

$$= L' \frac{\partial}{\partial t} \left[ \frac{\partial i}{\partial z} \right] \rightarrow \frac{\partial^2 i}{\partial z^2} + \frac{1}{L} u = 0,$$

where $q' = C'_s u [C \cdot m]$ represents the charge across the varactor. Inserting (5) into (6) yields the wave equation

$$\frac{\partial^2 q}{\partial z^2} - \frac{1}{L} u = 0,$$

in terms of voltage and capacitance charge. Abrupt junction varactors typically have a capacitance of the form $C(v) = \alpha (N/v + \varphi)^{n}$, where $\alpha$ is proportional to the area of the diode, $N$ is the doping level of the epitaxial layer, and $\varphi$ is the built-in potential. For small voltages, this law may be approximated in terms of charge by [6]:

$$q''(v) \equiv C'_s v - C'_s v^2,$$

where $C'_s[F \cdot m]$ is the linear capacitance and $C'_s[F \cdot m/V]$ is a coefficient of the nonlinear contribution [Fig. 1(b)]. Inserting (8) into (7), we obtain, after lengthy but straightforward development, the following voltage-wave partial-differential equation:

$$C'_s \frac{\partial^2 v}{\partial z^2} = 4C'_s \left[ \left( \frac{\partial v}{\partial t} \right)^2 + \frac{\partial v}{\partial t} \frac{\partial^3 v}{\partial z^3} + \frac{\partial v}{\partial z} \frac{\partial^3 v}{\partial z^2} \frac{\partial t}{\partial z} + \frac{1}{2} \frac{\partial^3 v}{\partial z^2} + \frac{1}{2} \frac{\partial^3 v}{\partial z^2} \frac{\partial t}{\partial z} \right] - \frac{1}{L} v = 0.$$
It can be verified that in the limit of a linear capacitance \( C_j^2 = 0 \), we retrieve the well known form of harmonic linear solutions

\[ v = v_0 \exp\left( \pm j(\omega t - kz) \right) \]

which, after insertion into Eq. (9), restores the dispersion relation \( \beta = -1/(\omega \sqrt{L C_j}) \) of Eq. (1).

4. HARMONIC AND PULSE WAVE RESPONSES

Usually, UWB systems use zero-DC pulses [8]. We consider here a bipolar Gaussian pulse input signal. The numerical results obtained with Agilent-HFSS® for a 50-cell artificial NL-LH-TH are presented in Figure 3. Figure 3(a) shows the \( S \) parameters of the TL for different values of the varactor capacitance [see Fig. 1(b)]. As long as the pulse spectrum lies above the highest cutoff frequency (smallest capacitance attained in the voltage swing), the signal will not be significantly altered by high-pass nature of the LH-TL. Figure 3(b) shows how a harmonic signal is distorted by the negative nonlinearity \( \gamma \) of the TL (see section 2). Finally, the response of the NL-LH-TL to a large magnitude pulse is shown in Figure 3(b).

5. CONCLUSION

The characteristics of NL-LH-TLs have been discussed for the first time. An NL-LH-TL was analyzed, and shown to exhibit anomalous frequency dispersion and negative nonlinearity. Its voltage-wave partial-differential equation was derived and its harmonic/pulse responses were described. Potential applications of NL-LH-TLs, such as pulse shaping for UWB and radar systems, were suggested.

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