Breakdown of the classical description of a local system

A well known story - with a twist

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Non-classical effects


In this article we demonstrate a genuine non-classical effect....

When is an effect truly non-classical?
Why important?

Quantum/classical transition  Is there a separation?

Quantum

Atoms  Superconducting circuits  Nanomechanical oscillators  Planets

We need criteria to test that something is non-classical

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What is not

- Discrete spectra
- Spontaneous emission
- Squeezing
- Continuous variable quantum teleportation

What is

Negative Wigner functions
Types of non-classicality

1. Agrees with quantum mechanics

2. The quantum description is different

3. Non-classical according to quantum mechanics

4. Violates any classical description

5. Bell inequalities
Agrees with quantum theory

True for planetary motion

\[ \langle \frac{\partial p}{\partial t} \rangle = -\langle \nabla V \rangle \]
Agrees with quantum theory

Discrete spectra

Absorption

Absorption of classical harmonic oscillator

$$\text{Abs} \propto \frac{\omega^2 \gamma}{(\omega_0 - \omega^2)^2 + \omega^2 \gamma^2}$$
Types of non-classicality

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The quantum description is different

Ex: Spontaneous emission

Dipole moment vanish \( \langle \hat{d} \rangle = 0 \)

No electric field \( \vec{E}(\vec{r}) = G(\vec{r})\langle \hat{d} \rangle = 0 \)

\[ \Rightarrow \text{No radiation} \]

Quantize:
\[ \hat{\vec{E}}(\vec{r}) = G(\vec{r}) d\sigma_- \]
\[ \hat{\vec{E}}^\dagger \hat{\vec{E}}(\vec{r}) = G(\vec{r})^2 d^2\sigma_+ \sigma_- \sim |e\rangle\langle e| \]
The quantum description is different

Harmonic oscillator with random phase

Dipole moment vanish \[ \langle d \rangle \sim d_0 \langle e^{i\phi} \rangle = 0 \]

Square of dipole does not \[ \langle d^*(t + \tau) d(t) \rangle \sim d_0^2 e^{i\omega \tau} \neq 0 \]

Radiation as before \[ \left\langle \hat{E}^\dagger \hat{E} \right\rangle = G(\vec{r})^2 d_0^2 \]

Bohr (1913): we need to do something to prevent atoms from radiating

Quantum effects

Ground state do not radiate even though \[ \langle \hat{d}(t + \tau) \hat{d}(t) \rangle \neq 0 \]

Rabi oscillation: phase lost during excitation
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Bell inequalities

Ideal test

Complications:

Requires two systems

Known Bell inequalities for continuous variables require complicated states

Also theory hard
Types of non-classicality

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Violates any classical description

Goal: convince somebody trained in classical physics that his/her view is wrong

Show there cannot be any classical description

Rule of the game:

Classical physics allowed
(==>weaker than Bell)

No quantum words allowed
Normal ordered products
Commutators etc.

J. C. Maxwell (1831-1879)
Squeezing

Squeezing: non-classical if fluctuations reduced below vacuum fluctuations

Well quantum mechanics tells us that every mode has fluctuations even vacuum

So this quantum thing is an initial condition?

No it is deeper than that

below what?

Well so you say
Squeezing

Classical theory?

Wigner function $W(x,p) \geq 0$ => Probability distribution

Gaussian operations + homodyne => Wigner function perfect classical description

Pick $x,p$ according to $W(x,p)$ and evolve

Non-classicality: picking $x,p$ wrong according to quantum mechanics

Same arguments to apply continuous variable quantum teleportation,......

Not bad science. Different objective.
Types of non-classicality

1. Agrees with quantum mechanics

2. The quantum description is different

3. Non-classical according to quantum mechanics
   Squeezing + homodyne

4. Violates any classical description

5. Bell inequalities

Genuine non-classical

Stronger criteria
Classical description

What is the most general description of a system?

Well it has a certain position and momentum

That is wrong in quantum mechanics

No, even that is wrong

Well it can have a distribution of course

Well prove it
Wigner functions

Grey background => quantum input (don’t tell Maxwell)

Single photon state => negative Wigner function
=> not a probability distribution

Have been done*:
- State reconstruction
- Maximum likelihood
- Inverse Radon

Quantum
Complicated, numerically unstable

Can we do something simple?

* Large fraction of audience et al
\[ \langle M^2(x, p) \rangle = \int dx dp W(x, p) M^2(x, p) \geq 0 \]

I agree, so let us measure \( x \) and \( p \) and see that it fits

Unfortunately I cannot measure both \( x \) and \( p \) but I can measure combination and infer \( W(x, p) \)


Picking the right function

Single photon state

\[ W(x, p) = \frac{1}{\pi} (1 - 2r^2) e^{-r^2} \]

\[ r^2 = x^2 + p^2 \]

Rotational symmetry

\[ M(x, p) = 1 + \sum_{n=1}^{N/2} C_{2n} r^{2n} \]

Pick \( M \) so that strong weight on center: \( \langle M^2 \rangle < 0 \)
Measuring higher orders

\[ l=2 \quad \langle r^4 \rangle = \langle (x^2 + p^2)^2 \rangle = \langle x^4 \rangle + \langle p^4 \rangle + 2\langle x^2p^2 \rangle \]

Measure “diagonal” quadratures

\[ \left\langle \left( \frac{x + p}{\sqrt{2}} \right)^4 \right\rangle + \left\langle \left( \frac{x - p}{\sqrt{2}} \right)^4 \right\rangle = \frac{1}{2}\left( \langle x^4 \rangle + \langle p^4 \rangle \right) + 3\langle x^2p^2 \rangle \]
Measure $2l$ quadratures: \[
\langle (x^2 + p^2)^l \rangle = \binom{2l}{l}^{-1} \frac{2^{2l}}{2^l} \sum_{m=1}^{2l} \langle Q_{\pi m/2l}^{2l} \rangle
\]

For any $C$s

I agree, so let us try it out.
Quantum expectation

Optimize $C$s => negative for $N \geq 4$ (requires 8 quadratures)
“Standard” photon subtraction experiments

Experiment

Fig. 1. (Color online) Setup diagram. The second harmonic generator (SHG) pumps the optical parametric oscillator (OPO). The filter cavities should allow only a single mode (at frequency $\omega - \omega_0$) to reach the single photon counting avalanche photo diode (APD). Two acousto optic modulators (AOM) shift the main frequency to $\omega - \omega_0$ and $\omega_0 + \omega_0$ – the latter is used for the local oscillator (LO) of the homodyne measurement, the former for an alignment beam, which is used to bring all cavities resonant with $\omega_0 - \omega_0$ but which is blocked during measurement.

The escape efficiency $\eta_{esc} = T / (T + L) = 0.97$. With an effective nonlinearity $E_{NL} \approx 0.020 \text{ W}^{-1}$, the threshold pump power for oscillation is around $P_{\text{thr}} = (T + L)^2 / 4 E_{NL} = 210 \text{ mW}$. The blue pump (430 nm) is generated by frequency doubling the main Ti:Sapph laser in a second harmonic generator (SHG) of similar geometry as the OPO, but with a KNO$_3$ crystal as the nonlinear medium. For single photon generation the pump should be rather weak to inhibit the population of higher photon numbers. The pumping strength is quantized as the pump parameter $\epsilon = \sqrt{P_b / P_{\text{thr}}}$, where $P_b$ is the blue pump power. This pump parameter is most easily inferred by observing the parametric gain, $G = 1 / (1 - \epsilon^2)$, of a beam of half the pump frequency seeded into the OPO.

The frequency spectrum of the OPO is illustrated in Fig. 2. With no seed beam, the output field in the degenerate frequency mode (half pump frequency) is quadrature-squeezed vacuum, whereas the non-degenerate modes taken individually are thermal states. They are, however, pairwise correlated symmetrically around the degenerate frequency. In the weak pump regime this means that for each down-converted photon in the $\omega - \omega_0$ mode one FSR below the degenerate frequency, there is a twin photon in the $\omega_0 + \omega_0$ mode one FSR above. In the time domain, the field operator correlations for the two modes are given by

$$\langle \hat{a}^{\pm}(t) \hat{a}^{\mp}(t') \rangle = \lambda^2 - \mu^2 / 4 (e^{-\mu |t - t'|^2 / 2} - e^{-\lambda |t - t'|^2 / 2})$$

$$\langle \hat{a}^{\mp}(t) \hat{a}^{\mp}(t') \rangle = \lambda^2 - \mu^2 / 4 (e^{-\mu |t - t'|^2 / 2} - e^{-\lambda |t - t'|^2 / 2})$$

(1)
“Standard” photon subtraction experiments

Homodyne detection with varying phase
  => Also works classically

Phase not locked => All quadratures the same

Cannot introduce violation
Violation by nearly 20 standard deviations.

\[
\frac{\langle M^2 \rangle}{\sigma_{M^2}}
\]
Conclusion

Non-classical: no classical description
(don’t assume quantum mechanics)

Simple strict non-classicality test

Can be violated on a single system using homodyne detection

Light field: one cannot assign a probability distribution to the position and moment - not even nature can know $x$ and $p$ simultaneously

I didn’t see that coming. I guess I will have to study this quantum thing.
Similar test should be applied to other macroscopic systems

Superconducting systems

Nanomechanical systems => this test works directly

Extension to Bell inequalities?
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Apologies to Maxwell

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