Telecommunication in a disordered environment with iterative time reversal

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Abstract
We present a method to transmit digital information through a highly scattering medium in a MIMO-MU (multiple input multiple output multiple users) context. It is based on iterations of a time-reversal process, and permits us to focus short pulses, both spatially and temporally, from a base antenna to different users. This iterative technique is shown to be more efficient (lower inter-symbol interference and lower error rate) than classical time-reversal communication, while being computationally light and stable. Experiments are presented: digital information is conveyed from 15 transmitters to 15 receivers by ultrasonic waves propagating through a highly scattering slab. From a theoretical point of view, the iterative technique achieves the inverse filter of propagation in the subspace of non-null singular values of the time-reversal operator. We also investigate the influence of external additive noise, and show that the number of iterations can be optimized to give the lowest error rate.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Recent research in the area of wave propagation in random media applied to telecommunications showed that, contrary to intuition, reverberation or scattering of waves in a disordered medium can actually help to increase the information transfer rate [1–5]. The key element therein is the ability of a communication system to exploit independent ‘channels’ of propagation.

In a homogeneous medium, the coherence of a wave is preserved as it propagates; for example, if a base antenna sends a pulse, two different users will receive highly correlated waveforms. In contrast, in a disordered environment, the spatial and temporal coherence are
affected by the inhomogeneities; as a result, two users apart by more than one coherence length will receive decorrelated waveforms. In a highly heterogeneous medium, wave propagation essentially depends on two parameters: the transport mean-free path $\ell^*$ and the diffusion constant $D$. Multiple scattering dominates when the thickness of the medium $L$ becomes much larger than $\ell^*$. In that case, the transmitted wave loses its spatial coherence and the transmitted field has a speckle pattern with a coherence length $\delta x \sim \lambda/2$ only limited by diffraction, i.e. two receivers apart by more than $\delta x$ will receive decorrelated waveforms. Moreover, the correlation frequency of a strong scattering medium is much smaller than that of a homogeneous medium. Physically, this means that the response (amplitude and phase) of a scattering medium to a sinusoidal excitation changes dramatically even if the frequency is changed only by a small amount, as if different frequencies were ‘seeing’ a different medium. The correlation frequency (sometimes referred to as the coherence bandwidth in telecommunication) is the frequency shift $\delta \omega$ so that $H(\omega)$ and $H(\omega + \delta \omega)$ are decorrelated, with $H(\omega)$ the frequency response of the medium. In a strong scattering medium with thickness $L$ and diffusion constant $D$, the typical correlation frequency is $\delta \omega \sim D/L^2$ [6, 7]. As a consequence, if the typical distance between users is larger than the coherence length $\delta x$, and the frequency bandwidth is larger than the correlation frequency $\delta \omega$, it is possible to take advantage of the disorder to increase the information transfer rate through all the independent ‘channels’ of the medium.

In that perspective, it was recently shown that a time-reversal antenna could efficiently focus a short pulse through a multiple scattering medium with a spatial resolution of the order of the wavelength [8]. Following this idea, Derode et al [9] proposed a communication scheme between a base antenna and a set of users to achieve a MIMO-MU (multiple input multiple output multiple users) communication. But, is time reversal the best technique? In this paper, we show that time reversal is not the optimal method but it can be modified in order to achieve the best focusing; additionally, the best communication method depends also on the random noise present in the channel.

Time reversal gives an elegant solution to focus a pulse at different points [10]; from a mathematical point of view, it is an approximation of the inverse filter [11]. Time reversal is a matched filter because it maximizes the ratio between the energy of the pulse at the focus and the total energy sent by the emission antenna. However, time reversal has two shortcomings that degrade the focusing quality in a practical case.

Firstly, time reversibility is broken in an absorbing medium [12, 13] and then the exact inversion is not guaranteed: the waveform that is recreated after time reversal and backpropagation is not the replica of the initial waveform. Second, a perfect time-reversal process requires the emission antenna to completely surround the propagation medium and to be sampled with a $\lambda/2$ pitch, thus forming the so-called time-reversal cavity [14]. As a complete time-reversal cavity is impossible to build, we usually work with a limited angular aperture and with non-ideal sampling which induce side lobes.

Is it possible to overcome these two problems of the classical time-reversal process? An alternative approach consists in measuring the complete set of responses between the base antenna and the users (which will be referred to as the propagation operator $H$) and computing explicitly the inverse operator $H^{-1}$ [12]. This technique permits us to focus a pulse with minimal temporal and spatial side lobes, even with a diffraction-limited system and an absorbing medium. However, it is a computationally demanding method that is not possible to implement in real time.

In this paper, we propose an iterative method based on successive time-reversal operations that converge to the optimal inverse filter of propagation. This technique has both the simplicity and real-time aspect of the time reversal and the precision of the inverse filter. We show that this iterative method can be applied to communication in highly scattering media.
In current communication systems, information is conveyed by electromagnetic waves in the GHz range. Since broadband time-reversal experiments with electromagnetic waves in this range are difficult to perform, we used ultrasonic waves in the MHz range in a highly multiple scattering medium to validate the method. For radiowaves in the GHz range, the typical wavelength is $\sim 10$ cm, and the scattering of waves on everyday life objects such as walls, cars, etc, can result in multipathing (fading); decay times of typically 1 $\mu$s (i.e. 1000 periods) in urban environments have been reported [15]. An ultrasonic experiment can reproduce this phenomenon at smaller scales: at 1.5 MHz in water, the wavelength is 1 mm and the spreading time is roughly 1 ms (i.e. 1500 periods). The ultrasonic experiment we report here is a way of mimicking the effect of strong scattering on a wave, whatever its nature is (electromagnetic, acoustic, elastic) and to illustrate how strong multiple scattering can not only be overcome but also taken advantage of to transmit more information.

In the second part, we present the basic idea of the iterative method and some preliminary results. In the third part we develop the theoretical basis of the method. Finally, the fourth part displays the results of telecommunication with ultrasonic waves through a complex multiple scattering medium. The effect of random additive noise is also investigated.

2. The iterative method

Figure 1 shows the experimental set-up. The base antenna is composed of 15 ultrasonic wideband transducers working at 1.5 MHz. The propagation medium is composed of a set of parallel steel rods of diameter 0.8 mm (density 29 rods cm$^{-2}$). The total thickness of the medium is 30 mm while its mean-free path $\ell^*$ is 3.5 mm, which was measured via the coherent backscattering effect [7]. Fifteen user antennas, each made of 1 transducer with the same characteristics than those of the base antenna, are placed at the opposite side of the medium.

Due to the strong multiple scattering in the medium, a short 1 $\mu$s pulse sent by the base antenna is received by the users as a long-lasting signal of typically 1 ms (see figure 2(a)). In this long impulse response, we cannot identify a direct wave and it is impossible to encode
Figure 2. (a) Typical signal received by one user after sending a 1 µs pulse by an element of the base antenna. Due to multiple scattering the signal lasts a very long time (1.5 ms). (b) Signal received by the same user after time reversal. (c) Signal received by another user after time reversal (interference noise).

information using the direct wave without considering the multiple paths of the waves inside the medium.

In a time-reversal communication process, the first step is to send a short pulse from one user; then the base antenna records the signal $h_j(t)$, $j = 1, \ldots, 15$ being the element of the base antenna. This signal is time-reversed and $h_j(-t)$ is sent back to the user. After propagation through the medium the user receives a short signal that, ideally, recreates the original pulse sent by the user antenna (see figure 2(b)). Due to the spatial decorrelation of the impulse responses in a multiple scattering medium, which yields a ‘super-resolution’ [8, 16], the other users do not receive any significant signal as we can see in figure 2(c). Indeed, during the forward step, the high spatial frequencies that would have been lost in a homogeneous medium are redirected to the base antenna, thanks to multiple scattering, and they can therefore be recreated by time reversal. As the time-reversed spherical wavefront emerges from the slab, its angular spectrum is much larger than the aperture of the base antenna itself. In other words, from a theoretical point of view, the multiple scattering medium plus the base antenna can be analysed as a virtual lens whose point spread function depends on the statistics of the scattering medium. This is discussed in detail in [16–18].

This method permits us to focus an elementary symbol (here, a short pulse) onto any user. Then it is possible to transmit more complex messages. For example, the BPSK (binary phase shift keying) modulation consists in sending a sequence of binary symbols $s_{j,k}$ at the instants $t_k$ with $s_{j,k} = \{-1 \text{ or } +1\}$ to the user $j$. To perform this communication, the base antenna has to send the signals

$$a_j(t) = \sum_{j,k} h_{ij}(-t + t_k)s_{j,k}$$  (1)
through the medium. Here, \( h_{ij}(t) \) is the impulse response between the user \( j \) and the element \( i \) of the base antenna. The users will receive a set of pulses whose sign correspond to the original message \( s_{j,k} \). Naturally, a higher density of information can be sent using standard coding techniques like m-PSK (m phase shift keying) [19].

Due to absorption in the medium and the finite aperture of the base antenna, time reversal is not able to recreate exactly the original pulse. A side lobe level of \(-20 \text{ dB} \) is present around the refocused pulse as we can see in figure 2(b). This deviation from the exact inverse filter of propagation is a limitation when we try to send dense information to all the users. When a large sequence of symbols is transmitted, the side lobes overlap which causes an inter-symbol interference that perturbs the communication.

The basic idea presented in this paper is to improve the time-reversal process by cleaning up the side lobes with an iterative process. This iteration consists of five steps.

The first step (figure 3(a)) is to focus a pulse on the user \( j_0 \) using a standard time-reversal process. We consider an objective \( o_j(t) \), \( j = 1, \ldots, n_u \) that is a pulse for \( j = j_0 \) and 0 elsewhere. This pulse must be a realistic objective with the same bandwidth as the transducers. A simple option is a sinc pulse \( o(t) = \sin(\pi t B) / \pi t B \), where \( B \) is the bandwidth of the transducers. We time-reverse this objective and send \( o_j(-t) \) from the user to the base antenna. The base antenna records the response \( e_i(t) \), \( i = 1, \ldots, n_b \) at each element \( i \). These signals are time-reversed and \( e_i(-t) \) is sent back to the users (figure 3(b)).

**Figure 3.** The iterative process: (a) and (b) standard time-reversal focusing; (c) sending the side lobes from the users to the base antenna; (d) focusing with the corrected signal.
Figure 4. (a) Time-reversal focusing on a user. The rms value of the side lobes is nearly 18 dB below the peak value. (b) Focusing on the same user using the iterative method: the side lobe level is now 33 dB below the peak value, but the peak value is 6 dB less than with standard time reversal.

All the users record the received signal \( r_j(t) \), \( j = 1, \ldots, n_u \). This signal is similar to the objective \( o_j(t) \) but shows unwanted side lobes that we want to cancel. The side lobes are calculated by a subtraction \( d_j(t) = o_j(t) - r_j(t) \) (figure 3(c)).

These signals \( d_j(t) \) are time-reversed and sent from the users to the base antenna. The base antenna records a signal \( c_i(t) \), \( i = 1, \ldots, n_b \).

If we send back the \( c_i(-t) \) from the base antenna to the users, we can obtain an approximation of the side lobes \( d_j(t) \) that we want to eliminate. Sending \( c_i(t) + c_i(t) \) permits us to diminish the side lobes when we focus on one user.

This process can be iterated from step 2 in order to completely eliminate the side lobes.

This method only needs a standard time-reversal process and a signal subtraction, so it is easy to implement in a real-time machine and does not demand heavy, and possibly unstable, computations.

In figure 4 we can see an example of a focused wave with a standard time-reversal process and after 15 steps of the new iterative method. The temporal side lobes were reduced by 15 dB in this example. The maximal interferences on the other users were reduced between 13 dB and 8 dB (see figure 5).

However, the disadvantage of this iterative method is that the amplitude of the focused pulse is significantly lower than in a standard time-reversal focusing. Theoretically, time reversal is a matched filter and then it gives the maximal amplitude at the focus for a fixed emission energy. The iterative method tries to compensate the ‘holes’ of the spectrum by increasing the emission energy at the frequencies where the medium is more opaque, this equalization process decreases the focusing amplitude for a fixed emission energy. Figure 4 shows that the amplitude of the pulse recreated by the iterative method is 6 dB below the value obtained by a standard time reversal. Therefore, if an external noise is added, the signal-to-noise ratio would be decreased. In that case, a compromise has to be found between the peak amplitude and the reduction of the side lobes; this will be analysed in the fourth part.
Performing this iterative process for each user antenna, we obtain a set of emission signals $e_{i,j}(t)$ where the index $j = 1, \ldots, n_u$ is the user and the index $i = 1, \ldots, n_b$ refers to an element of the base antenna.

Similar to equation (1), we can encode information sequences $s_{j,k}$ by making the base antenna send

$$a_i(t) = \sum_{j,k} e_{ij}(-t + t_k)s_{j,k}.$$  

(2)

In figure 6 we can see the result of sending a random set of binary symbols (+1/-1) to each user with an inter-symbol separation of 1.5 $\mu$s with time reversal (equation (1)) and the iterative method (equation (2)). The symbols are detected using a standard matched filter in reception. In the case of time reversal, there is inter-symbol interferences (figure 6(b)) and one of the transmitted bits is wrong. Using the iterative method, this interference is removed (figure 6(c)). For a communication at a transfer rate of 0.5 Mbit s$^{-1}$, an experimental measure of the transfer error rate (defined as wrong_bits/total_bits) gives 12% for standard time reversal and 0.6% for the iterative method. This example shows the interest of the iterative filter in order to reduce the transfer error due to interferences between the symbols, as well as between the users. Of course, more complex modulations can also be implemented and we will present some of them in the last part of the paper.

3. Theoretical background

We consider a system composed of a base antenna of $n_b$ elements and $n_u$ users dispersed in an inhomogeneous medium. The propagation between the base antenna and the users can be described by the set of impulse responses $h_{i,j}(t), i = 1, \ldots, n_b, j = 1, \ldots, n_u$ relating the $i$th element of the base antenna to the user $j$. This set of impulse responses is called the propagation operator.

If we send a signal $e_i(t), i = 1, \ldots, n_b$ from the base antenna we can calculate the signal $r_j(t), j = 1, \ldots, n_u$ obtained at the user antennas as

$$r_j(t) = \sum_i h_{ij}(t) \otimes e_i(t).$$  

(3)
Figure 6. Example of a BPSK modulation with the standard time-reversal focusing and with the iterative method. (a) The original message is composed of a matrix of 300 by 15 bits to be sent to the 15 users (only a section of 10 bits for the user 8 is shown in the figure). (b) Received signal with a time-reversal process. After the detection of the sign of the pulse we recover the message. Due to the interference side lobes the pulses are not clearly separated and the bit marked with an arrow is wrong. (c) Received signal with the iterative method. The pulses are clearly separated, no errors were detected.

Else, in the Fourier domain

\[ R_j(\omega) = \sum_i H_{ij}(\omega)E_i(\omega) \]  

that may be written in matrix form as

\[ R(\omega) = H(\omega)E(\omega). \]  

If we send a set of signals \( c_j(t), j = 1, \ldots, n_u \) from the user antennas we will obtain a signal \( d_i(t), i = 1, \ldots, n_b \) at the base antenna related to \( c_j(t) \) by the transposed matrix \( H^T \)

\[ D(\omega) = H(\omega)C(\omega) , \]  

\( H \) describes the propagation from the base to the users and \( H^T \) the propagation from the users to the base.

Our objective is to focus a short pulse on the \( m \)th user, an objective that we can write as \( o(t) = [0, \ldots, \delta(t), \ldots, 0] \) or in the Fourier domain \( O(\omega) = [0, \ldots, 1, \ldots, 0] \).

A time-reversal focusing begins by sending \( o(-t) \) from the user antenna. \( o(-t) \) is in the Fourier domain \( O^*(\omega) \) where the * denotes the complex conjugate, then we can calculate the signals received on the base antenna after the propagation as

\[ E(\omega) = H(\omega)O^*(\omega). \]
We time-reverse this signal and we send \( E^*(\omega) \) back to the user antennas; the signal \( F(\omega) \) received after the propagation is

\[
F(\omega) = H(\omega)E^*(\omega) = H(\omega)H^\dagger(\omega)O(\omega)
\]  

(8)

where \( \dagger \) denotes the transpose conjugate: \( H^\dagger = \dagger H \). Thus, the focusing quality obtained by standard time-reversal processing is directly linked to the properties of the operator \( [9] \) \( \Delta(\omega) = H(\omega)H^\dagger(\omega) \), the so-called time-reversal operator. If the system consists in a non-dissipative closed cavity and if the set of transmitters fully maps the boundary surface of the cavity, a time-reversal operation will tend to recreate the inverse propagation operator of the system,

\[
\Delta = H(\omega)H^\dagger(\omega) \cong I(\omega) \quad H^{-1}(\omega) \cong H^\dagger(\omega) \quad h_{ji}^{-1}(t) \cong h_{ji}(-t)
\]  

(9)

where \( I(\omega) \) is the identity operator.

Nevertheless, as soon as information losses occur during a time-reversal experiment (for example, in an open or a lossy system) time reversal is not able to fully achieve the inverse of the propagation any more. In such situations, it is not ensured that time reversal will be able to achieve a good focusing quality. The technique that we propose tries to iterate experimentally time-reversal operations in order to improve the focusing result and finally recover the optimal focusing achieved by inverse filtering.

As we have seen in section 2, the first step of this method is the standard time-reversal focusing described before. We choose an objective \( O(\omega) \), this objective is time-reversed and \( O^*(\omega) \) is sent from the user antennas. After propagation, the waveform received on the base antenna is

\[
E_1(\omega) = \dagger H(\omega)O^*(\omega).
\]  

(10)

These signals are time-reversed and sent from the base antenna to the users. The signals recorded on the users can be written as

\[
R_1(\omega) = H(\omega)E_1^*(\omega).
\]  

(11)

The difference between this focusing result and the desired focusing objective is

\[
D_1(\omega) = O(\omega) - R_1(\omega).
\]  

(12)

This wavefield is then time-reversed and transmitted by the user antennas. The resulting wavefield propagates through the medium and \( \dagger H(\omega)D_1^*(\omega) \) is recorded on the base antenna. \( \dagger H(\omega)D_1^*(\omega) \) can be seen as a ‘correcting’ signal. If we time-reverse this signal and send it to the users it will give a good approximation of the difference \( D(\omega) \). Then we build a new emission signal

\[
E_2(\omega) = E_1(\omega) + \dagger H(\omega)D_1^*(\omega).
\]  

(13)

After time-reversal emission and propagation, the new focusing result on the users will be

\[
R_2(\omega) = H(\omega)E_2^*(\omega).
\]  

(14)

This process can be reiterated in order to correct the emission signals \( E_n(\omega) \) step by step,

\[
\begin{cases}
R_n = HE_n^* \\
D_n = O - R_n \\
E_{n+1} = E_n + \dagger HD_n^*.
\end{cases}
\]  

(15)
with an initial step $E_1(\omega) = (\mathbf{H}(\omega))^{*}O(\omega)$. At this point, the basic problem is to determine whether this iteration converges to the solution. Note that for the sake of clarity, we suppress the $\omega$ dependence in the following equations. As we are only interested in $R_n$, we can write the iteration process (15),

$$R_{n+1} = R_n - \Delta (R_n - O).$$

(16)

It is easy to demonstrate that the $n$th iteration of (16) is

$$R_n = [I - (I - \Delta)^n]O,$$

(17)

where $I$ is the identity matrix. The difference between the focusing $R_n$ at the $n$th iteration and the initial focusing objective $O$ is

$$D_n = (I - \Delta)^n O.$$

(18)

This difference must converge to 0 in an exact focusing. In order to analyse the convergence of $D_n$, we consider the diagonalization of $\Delta$,

$$\Delta = \mathbf{PSP}^\dagger,$$

(19)

where $\mathbf{S}$ is a real and diagonal matrix containing the eigenvalues $\lambda_i = 1, \ldots, n_u$ of the time-reversal operator in a decreasing order, and $\mathbf{P}$ is a unitary matrix. The set of eigenvalues can be divided into two parts: the $M$ non-null eigenvalues, which describe the physically relevant data contained in the propagation operator, and the null ones. Thus, this number $M$ will depend on all the parameters defining the experimental set-up: the number of elements of the base antenna, the number of users, the complete geometry of the system and the nature of the propagating medium. As an example, if the number of users $n_u$ is greater than the number of elements of the base antenna $n_b$, the rank of the null eigenvalues’ subspace will be at least the difference $n_u - n_b$. This number of null eigenvalues can also increase when two or more elements are very close. In that particular situation, the impulse responses of two neighbours are strongly correlated and similar columns appear in the operator $\mathbf{H}$ giving rise to null eigenvalues in $\Delta$.

How do null and non-null eigenvalues behave respectively during the iterative process? According to equations (18) and (19) the $n$th difference will be

$$D_n = \mathbf{P}(1 - \mathbf{S})^n \mathbf{P}^\dagger O.$$

(20)

The eigenvalues of $(1 - \mathbf{S})^n$ are $(1 - \lambda_i)^n$, and they converge to 0 if $0 < \lambda_i < 2$. Thus, the difference between the focusing objective and the focusing pattern obtained at iteration $n$ will tend to zero if all the eigenvalues lie between 0 and 2. As $\Delta$ is hermitic and positive defined, its eigenvalues are real and positive. Besides, by normalizing adequately\footnote{At each iteration we record, amplify and re-send the signals; by changing this amplification factor we can adjust the normalization of the propagation matrix.} the propagation operator $\mathbf{H}$ we can easily force all the eigenvalues to be between 0 and 2. In other words, if the eigenvalues are different from zero, the difference $D_n$ will tend to zero during the iterative process. In contrast, the null eigenvalues will remain unchanged during the iterative process. Thus, we can decompose the eigenvalues’ space into two distinct subspaces corresponding to the non-null eigenvalues’ subspace and the null eigenvalues’ subspace. So, the iterative
process allows us to project the focusing objective $O$ over the non-null subspace, A residual $D_\infty$ will remain, corresponding to the projection of the objective over the null subspace.

$$D_\infty = \mathbf{P} \begin{bmatrix} 0 & & & \vdots & 0 \\ & 0 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix} \mathbf{P}^\dagger O.$$  \hspace{1cm} (21)

As the iterations go on, this method permits a progressive transition from the time-reversal focusing to the inverse filter focusing. We will use later this idea of an ‘intermediate’ method between time reversal and the inverse filter to optimize the communication in a noisy environment.

4. Experimental results

Using the system described in figure 1, we present here a comparison between communications using either classical time reversal or the iterative method. We implemented a PSK (phase shift keying) modulation of 1, 2 and 3 bits per symbol. This modulation codifies the information in the phase of the signal, for example in a 1 bit BPSK (binary PSK) codification one of the symbols is codified with phase 0 and the other with phase $\pi$, in a 4-PSK the 4 combinations of the 2 bits are codified with phases $0, \pi/2, \pi$ and $3\pi/2$. The focusing signals for the iterative method were obtained after 15 iterations.

Figure 7 shows the error rate for different transfer rates. The probability of error is calculated as $P_{\text{error}} = \frac{\text{wrong bits}}{\text{total bits}}$; each time we transmitted as many bits as necessary to reach a statistically meaningful result. The iteration gives a better result than the time-reversal method, particularly for low transfer rates.

We can deduce a simple model that accounts for the shape of these graphics. When we focus a symbol on a user, it is perturbed by all the other symbols sent to the same user and to the other users. Each of these symbols generates an interference noise $\eta_i$, with $i$ the index of symbols. The total noise received is

$$\eta_{\text{int}} = \sum_{i=1}^{N_{\text{sym}}} \eta_i$$  \hspace{1cm} (22)

and $N_{\text{sym}}$ is the total number of interfering symbols.

The $\eta_i$ are independent noises, supposing they have the same variance $\tilde{\sigma}_{\text{int}}^2$, the total variance of $\eta_{\text{int}}$is $\sigma_{\text{int}}^2 = \tilde{\sigma}_{\text{int}}^2 N_{\text{sym}}$.

The total number of symbols $N_{\text{sym}}$ that interfere is $N_{\text{sym}} = n_u \tau / \Delta$, $n_u$ being the number of users, $\tau$ the duration of the impulse response from the base antenna to the users and $\Delta$ the time interval between two successive symbols. The quantity $\tau / \Delta$ measures the number of symbols whose temporal side lobes overlap (two bits separated more than $\tau$ cannot interfere). In conclusion, the dispersion of the total error is

$$\sigma_{\text{int}} = \tilde{\sigma}_{\text{int}} \sqrt{n_u \tau / \Delta}.$$  \hspace{1cm} (23)

In the simplest case of a BPSK modulation, the probability of having an error in the communication is $P_{\text{error}} = P(\eta_{\text{int}} > 1)$, in this case a negative bit is wrongly detected as +1, and vice versa. If the number of interfering symbols $N_{\text{sym}}$ is large, we can assume a normal distribution of $\eta_{\text{int}}$ and the error probability can be calculated as

$$P_{\text{error}} = \frac{1}{2} \text{erfc} \left( \frac{1}{\sigma_{\text{int}} \sqrt{2}} \right).$$  \hspace{1cm} (24)
For an \( m \)-PSK codification a good approximation of the probability of error is [19]

\[
P_{\text{error}} = \frac{1}{\log_2 m} \text{erfc} \left( \frac{\sqrt{\log_2 m \sin(\pi/m)}}{\sigma_{\text{int}} \sqrt{2}} \right).
\]  

(25)

For a 4-PSK modulation, the probability of error is the same as for BPSK, for a 8-PSK the error is higher than for a 4-PSK. It is important to note that this degradation of the 8-PSK is a problem of the PSK codification and not of the time reversal or the method used to focus the symbol on the user [19]. Figure 7 shows that this simple model predicts the correct value of the error. The parameter \( \tilde{\sigma}_{\text{int}} \) was calculated by focusing a pulse and measuring the standard deviation of the amplitude of the interferences in a time \( \tau \). The time \( \tau \) must be large enough to include all the physical signals but an overestimation is not critical because it decreases \( \tilde{\sigma}_{\text{int}} \) and it is compensated by the augmentation of \( \tau \) in equation (23).

4.1. Influence of external noise

The previous calculations were carried out without random noise and only dealt with errors due to interferences between different users and between different symbols sent to the same user. They showed how efficient the iterative method is to clean up these unwanted interferences.

Figure 7. Error rate for different speeds of communication using either time reversal (TR) or the iterative method (IT). The dots are the experimental data and the lines are the result of the theoretical model. (a) BPSK modulation, (b) a 4-PSK modulation, (c) a 8-PSK modulation. For all cases, the iterative method gives a better result than standard time reversal.
Real communications are always exposed to random noise perturbations. We can understand its effect taking into account random noise in the model of equation (24). If we assume a Gaussian random noise of standard deviation $\sigma_{\text{noise}}$ independent from the information to be sent, the total deviation will be

$$\sigma_{\text{total}} = \sqrt{\sigma_{\text{int}}^2 + \sigma_{\text{ext}}^2}.\quad (25)$$

For a BPSK communication, then the total error in the presence of random noise is

$$P_{\text{error}} = \frac{1}{2} \text{erfc}\left(\frac{1}{\sqrt{2(\sigma_{\text{int}}^2 + \sigma_{\text{ext}}^2)}}\right).\quad (26)$$

In this case, as we have mentioned earlier, the comparison between time reversal and the iterative method is more delicate: a compromise has to be found between time reversal (that gives the highest focusing amplitude but bad side lobes) and the iterative method (that gives the lowest side lobes but decreases the amplitude of the focused peak). We can illustrate this problem, considering two extreme cases:

1. A communication without external noise. In this case the best solution is to minimize the interference noise $\sigma_{\text{int}}$ and the best result is given by the inverse filter (i.e. the iterative method).

2. A communication with a high external noise. In this case $\sigma_{\text{total}}$ is dominated by the external noise and then the best solution is to maximize the signal/noise ratio for the focused pulse.

As time reversal optimizes the amplitude of the focusing pulse (for a given amount of transmitted energy) it gives a better result than the iterative method.

For an intermediate noise level, we need to use an ‘intermediate’ method between the time reversal and the iterative method that gives the best compromise between amplitude and side lobes: we can try and find out at which iteration the iterative method should be stopped to give the best compromise.

The concrete problem we want to solve is to find the number of iterations that minimizes the total noise dispersion $\sigma_{\text{tot}}$ of equation (25). In figure 8(a) we measured the amplitude of the focusing pulse $A$ versus the number of iterations $A = A(n)$. As time reversal maximizes the amplitude, we normalized the amplitudes in order to have 1 for the time-reversal focusing. Figure 8(b) shows the dispersion of the side lobes $\tilde{\sigma}_{\text{int}}$ versus the number of iterations. Finally (Figure 8(c)) we can deduce from the two previous curves the dispersion of the side lobes versus the amplitude $\tilde{\sigma}_{\text{int}} = \tilde{\sigma}_{\text{int}}(A)$. A third-order polynomial interpolation is calculated from the experimental results in order to have a simplified numerical model to make the following calculations.

The interference noise $\sigma_{\text{int}}$ is calculated from equation (23),

$$\sigma_{\text{int}} = v_{\text{int}}(A)\sqrt{n_u \tau / \Delta}.\quad (27)$$

If we assume a Gaussian external noise of dispersion $v_{\text{ext}}$, the dispersion of the noise before the detection is

$$\sigma_{\text{ext}} = v_{\text{ext}}/A.\quad (28)$$

substituting (27) and (28) in (25) we have

$$\sigma_{\text{total}}(A) = \sqrt{\tilde{\sigma}_{\text{int}}^2(A)(n_u \tau / \Delta) + v_{\text{ext}}^2/A^2}.\quad (29)$$

In figure 9 we calculated $\sigma_{\text{tot}}(A)$ for different values of the external noise $v_{\text{ext}}$, the other parameters were $n_u = 15$, $\tau = 1.5$ ms and $\Delta = 1.5$ $\mu$s. The minimal value of $\sigma_{\text{tot}}$ gives the optimal value of the amplitude and, going to figure 8(b), we can obtain the optimal number of iterations.
Figure 8. (a) Interference noise level versus the number of iterations. The noise decreases monotonically with the number of iterations. (b) Peak amplitude versus the number of iterations. Standard time-reversal focusing (1 iteration) gives the maximum amplitude. (c) Interference noise level versus peak amplitude. The line is an interpolation with a third-order polynomial.

Figure 9. Choosing the optimal number of iterations when the communication is perturbed by external noise. The lines show the total noise dispersion versus the focusing amplitude for external noises level $\sigma_{ext}$ of 0.1, 0.2, 0.3, 0.4 and 0.5. Each curve has a minimum value marked with a triangle. The number over the triangle is the number of iterations to reach this minimal noise level.

In figure 10 we can see an experimental verification of this optimal point. With an external noise of 0.3, there is a minimum of the error rate at nearly 9 iterations which is the optimal value predicted in figure 9.
5. Conclusions

We presented a method to focus short pulses, both spatially and temporally, on different users through a scattering medium. This method uses a stable iterative time-reversal process to recreate an arbitrary focusing objective on one user, minimizing the spatial and temporal side lobes. From a theoretical point of view, the iteration achieves the inverse filter of propagation in the subspace of non-null singular values of the time-reversal operator. The iteration follows a simple scheme based on time reversal and signal subtraction operations and does not need heavy computation.

We proposed to apply this method to telecommunications in complex environments and we presented experiments in the ultrasonic MHz range in a multiple scattering medium. In this case, the iterative method is able to decrease the interference side lobes by 13 dB; the probability of inter-symbol interferences is lower, which permits a faster error-free communication. A simple model was developed to predict the transmission error rate.

However, in the presence of external noise a compromise has to be found between the necessity of increasing the signal-to-noise ratio for a given amount of transmitted energy (which is best performed by standard time-reversal) while diminishing inter-symbol interferences (which is minimized by the inverse filter). This compromise can be found by using a finite number of iterations, which constitutes an ‘intermediate method’ between classical time reversal and the inverse filter.

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References


